Defn: Let \( \{U_1, \ldots, U_n\} \) be a finite indexed open cover of \( X \). An indexed family of continuous functions 
\[
\phi_i : X \to [0, 1]
\]
is a partition of unity dominated by \( \{U_1, \ldots, U_n\} \) if
1) support \( \phi_i \subset U_i \) for all \( i \).
2) \( \sum_{i=1}^{n} \phi_i(x) = 1 \) for all \( x \).

Ex: \( f_i : \mathbb{R} \to [0, 1], f_i(x) = \frac{1}{2} \) is a partition of unity dominated by \( U_i = \mathbb{R}, i = 1, 2 \)

Ex: \( \phi_i : \mathbb{R} \to [0, 1], \)
\[
\phi_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}, \quad \phi_2(x) = \begin{cases} 1 & \text{if } x < 0 \\ 1 - x & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}
\]
is a partition of unity dominated by 
\( U_1 = (-1, \infty), \quad U_2 = (\infty, 2) \)

Note: partition of unity for an arbitrary open cover will be defined in section 41 (one more condition, which finite covers automatically satisfy, will be needed).

Thm 36.1: (Existence of finite partitions of unity): Suppose \( X \) is \( T_4 \) and \( X \subset \bigcup_{i=1}^{n} U_i^{\text{open}} \). Then there exists a partition of unity dominated by \( \{U_1, \ldots, U_n\} \)

Thm 36.2: \( X \) compact \( m \)-mfld, then \( X \) can be imbedded in \( \mathbb{R}^N \) for some \( N \in \mathbb{Z} \).

Section 39:
A collection \( \mathcal{A} \) of subsets of \( X \) is locally finite if for all \( x \in X \), there exists \( U \) open such that \( x \in U \) and \( U \) intersects only finitely many elements of \( \mathcal{A} \).

Ex: \( \mathcal{A} = \{(n, n + 2) \mid n \in \mathbb{Z}\} \) is locally finite.

Ex: \( \mathcal{C} = \{(n, n + 2) \mid n \in \mathbb{Z}_+\} \) is locally finite.

Ex: \( \mathcal{D} = \{(0, n) \mid n \in \mathbb{Z}_+\} \) is NOT locally finite.

Ex: A finite collection of sets is locally finite.

The indexed family \( \{A_\alpha \mid \alpha \in J\} \) is a locally finite indexed family in \( X \) if for all \( x \in X \), there exists \( U \) open such that \( x \in U \) and \( U \) intersects \( A_\alpha \) for only finitely many \( \alpha \).

Ex: If \( A_i = \mathbb{R} \) for all \( i \in \mathbb{Z} \), then \( \{A_i \mid i \in \mathbb{Z}\} \) is NOT a locally finite indexed family in \( X \), but \( \{A_i \mid i \in \mathbb{Z}\} \), as a collection of set(s), is locally finite (since it contains only one set).