Defn A is *dense* in X is $\overline{A} = X$. X is *separable* is separable if it has a countable dense subset.

Ex:

Thm 30.3b: 2nd countable implies separable.

Proof: Let $\{B_n \mid n \in \mathbf{N}\}$ be a countable basis for X. Let $A =$

Defn: X is Lindelof if $X \subset \cup_{a \in A} U_a$, U_a open implies there exists $a_i \in A$ such that $X \subset \cup_{i=1}^{\infty} U_{a_i}$.

Thm 30.3a: 2nd countable implies Lindelof.

Proof: Let $\{B_n \mid n \in \mathbf{N}\}$ be a countable basis for X. Suppose $X \subset \cup_{a \in A} U_a$, U_a open

$\mathcal{R}_{\mathcal{L}} = \mathcal{R}$ with the lower limit topology is

Separable:

1st countable:

Not 2nd countable.

Lindelof.

Step 1: Every open covering has a countable subcover iff every open covering of basis elements has a countable subcover

$\mathcal{R}_{\mathcal{L}} \times \mathcal{R}_{\mathcal{L}}$ is not Lindelof.

Hence the product of Lindelof spaces need not be Lindelof.

A subspace a of Lindelof space need not be Lindelof.

Ex: The ordered square $= [0,1] \times [0,1]$ with the order topology using the dictionary order is Lindelof, but the subspace $[0,1] \times (0,1)$ is not.