Local Properties:

Defn: $X$ is first countable if $X$ has a countable basis at each of its points.

Defn: $X$ is locally connected at $x$ if for every neighborhood $U$ of $x$, there exists connected open set $V$ such that $x \in V \subset U$.

$X$ is locally connected if $X$ is locally connected at each of its points.

Defn: $X$ is locally path connected at $x$ if for every neighborhood $U$ of $x$, there exists path connected open set $V$ such that $x \in V \subset U$.

$X$ is locally path connected if $X$ is locally path connected at each of its points.

29. Local Compactness

Defn: $X$ is locally compact at $x$ if there exists a compact set $C \subset X$ and a set $V$ open in $X$ such that $x \in V \subset C$.

$X$ is locally compact if it is locally compact at each of its points.

Examples
1. $\mathbb{R}^n$ with the usual topology is locally compact,
2. $\mathbb{R}$ w/ lower limit topology is NOT locally compact.

Thm 29.1: $X$ is locally compact Hausdorff iff and only if there exists a $Y$ such that
1.) $X$ is a subspace of $Y$
2.) The set $Y - X$ consists of a single point.
3.) $Y$ is a compact Hausdorff space.

Moreover if $Y$ and $Y'$ both satisfy these conditions, then there is a homeomorphism of $Y$ and $Y'$ that equals the identity on $X$.

I.e., $X$ is locally compact Hausdorff iff it has a one-point compactification.

Lemma 29.3: Suppose $X$ is locally compact.
If $A$ is closed in $X$, then $A$ is locally compact.
If $X$ is Hausdorff and $A$ is open in $X$, then $A$ is locally compact Hausdorff.

Cor 29.4: $X$ is homeomorphic to an open subspace of a compact Hausdorff space iff $X$ is locally compact Hausdorff.