Defn: X is locally connected at x if for every neighborhood U of x, there exists connected open set V such that $x \in V \subset U$.

X is locally connected if x is locally connected at each of its points.

Defn: X is locally path connected at x if for every neighborhood U of x, there exists path connected open set V such that $x \in V \subset U$.

X is locally path connected if x is locally path connected at each of its points.

29. Local Compactness

Defn: X is locally compact at x is there exists a compact set $C \subset X$ and a set V open in X such that $x \in V \subset C$. $\breve{}X$ is locally compact if it is locally compact at each of its points.

Examples $\breve{1}$.) R^n with the usual topology is locally compact, $\breve{2}$.) R with the lower limit topology is not locally compact.

Thm 29.2: Suppose X is Hausdorff. Then X is locally compact if and only if for all $x \in X$ and for every neighborhood U of X, there is an open set V such that $\overline{V}$ is compact and $x \in X \subset \overline{V} \subset U$.

Thm 29.1: X is locally compact Hausdorff iff and only if there exists a Y such that $\breve{1}$.) X is a subspace of Y $\breve{2}$.) The set $Y - X$ consists of a single point. $\breve{3}$.) Y is a compact Hausdorff space.

Moreover if Y and Y' both satisfy these conditions, then there is a homeomorphism of Y and Y' that equals the identity on X.