24. Connected Subspaces of the Real Line.

Defn: A simply ordered set L having more than one element is called a **linear continuum** if the following hold:
(1) L has the least upper bound property.
(2) If $x < y$, there exists z such that $x < z < y$

Thm 24.1: If L is a linear continuum in the order topology, then L is connected, and so are intervals and rays in L.

Cor 24.2: The real line is connected and so are intervals and rays in $\mathcal{R}$.

Thm 24.3 (Intermediate value theorem). Let $f : X \to Y$ be a continuous map where X is connected and Y is an ordered set with the order topology. If $a, b \in X$ and if $r \in Y$ is a point lying between $f(a)$ and $f(b)$, then there exists a point c of X such that $f(c) = r$.

Defn: Given points $x, y \in X$, a **path** in X from x to y is a continuous map $f : [a, b] \to X$ such that $f(a) = x$ and $f(b) = y$.

A space is **path connected** if every pair of points of X can be joined by a path in X.

Lemma: A path connected space is connected.

Lemma: There exists a connected space which is not path connected.