$\mathcal{R}P^n = (\mathcal{R}^{n+1} - \{0\})/(\mathbf{x} \sim t\mathbf{x})$.

Ex: $\mathcal{R}P^1 = (\mathcal{R}^2 - \{0\})/(\mathbf{x} \sim t\mathbf{x})$

Let $p : \mathcal{R}^2 - \{0\} \to (\mathcal{R}^2 - \{0\})/(\mathbf{x} \sim t\mathbf{x})$, $p(\mathbf{x}) = [\mathbf{x}]$ and give $\mathcal{R}^2 - \{0\})/(\mathbf{x} \sim t\mathbf{x})$ the quotient topology.

Take one representative per equivalence class and determine g:

$$g : \mathcal{R}^2 - \{0\} \to S^1,\ g(re^{i\theta}) = e^{2i\theta}.$$

Define $f : (\mathcal{R}^2 - \{0\})/(\mathbf{x} \sim t\mathbf{x}) \to S^1$, $f([\mathbf{x}]) = g(\mathbf{x})$ (per thm 22.2).

$$\begin{array}{ccc} \mathcal{R}^2 - \{0\} & & \\ p \downarrow & & \\ (\mathcal{R}^2 - \{0\})/(\mathbf{x} \sim t\mathbf{x}) & \dashrightarrow & S^1 \end{array}$$

Note $(f \circ p)(\mathbf{x}) = g(\mathbf{x})$.

Note g is onto and continuous by calc 1.

Note g is an open map.

Hence g is a quotient map.

Thus by Thm 22.2, f is a homeomorphism.