Thm 19.1, 2: Comparison of box and product topologies. Let $\mathcal{B}_\alpha$ be a basis for $X_\alpha$

Basis for the box topology: $\{\prod U_\alpha \mid U_\alpha \text{ open in } X_\alpha \}$

or $\{\prod B_\alpha \mid B_\alpha \in \mathcal{B}_\alpha \}$

Basis for the product topology:

$\{\prod U_\alpha \mid U_\alpha \text{ open in } X_\alpha \}$,

$U_\alpha = X_\alpha$ for all but finitely many $\alpha$

or $\{\prod B_\alpha \mid B_\alpha \in \mathcal{B}_\alpha, \ i = 1, \ldots, n, \ B_\alpha = X_\alpha \text{ for } \alpha \neq \alpha_i, \ i = 1, \ldots, n \}$

Hence box topology is finer then the product topology.

Thm 19.3: Let $A_\alpha$ be a subspace of $X_\alpha$. Then $\Pi A_\alpha$ is a subspace of $\Pi X_\alpha$ if both products are given the box topology or if both products are given the product topology.

Thm 19.4: If $X_\alpha$ is Hausdorff for all $\alpha$ then $\Pi X_\alpha$ is Hausdorff in both the box and product topologies.

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Thm 19.5: $\overline{\Pi A_\alpha} = \overline{\Pi A_\alpha}$ in both the box and product topologies.

Thm 19.6: Suppose $f_\alpha : X \to Y_\alpha$. Define $f : X \to \Pi_{\alpha \in A} Y_\alpha$ by $f(x) = (f_\alpha(x))_{\alpha \in A}$. Let $\Pi X_\alpha$ have the product topology. Then $f$ is continuous is and only if $f_\alpha$ is continuous $\forall \alpha$. 