Thm 19.1, 2: Comparison of box and product topologies. Let $\mathcal{B}_\alpha$ be a basis for X_α

Basis for the box topology: $\{\Pi U_\alpha \mid U_\alpha \text{ open in } X_\alpha\}$
or $\{\Pi B_\alpha \mid B_\alpha \in \mathcal{B}_\alpha\}$

Basis for the product topology:

$$\{\Pi U_\alpha \mid U_\alpha \text{ open in } X_\alpha, \quad U_\alpha = X_\alpha \text{ for all but finitely many } \alpha\}$$

or $$\{\Pi B_\alpha \mid B_{\alpha_i} \in \mathcal{B}_{\alpha_i},\ i = 1, ..., n, \quad B_\alpha = X_\alpha \text{ for } \alpha \neq \alpha_i,\ i = 1, ..., n\}$$

Hence box topology is finer then the product topology

Thm 19.3: Let A_α be a subspace of X_α. Then ΠA_α is a subspace of ΠX_α if both products are given the box topology or if both products are given the product topology.

Thm 19.4: If X_α is Hausdorff for all α then ΠX_α is Hausdorff in both the box and product topologies.

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Thm 19.5: $\Pi \overline{A_\alpha} = \overline{\Pi A_\alpha}$ in both the box and product topologies.

Thm 19.6: Suppose $f_\alpha : X \to Y_\alpha$. Define $f : X \to \Pi_{\alpha \in A} Y_\alpha$ by $f(x) = (f_\alpha(x))_{\alpha \in A}$. Let ΠX_α have the product topology. Then f is continuous is and only if f_α is continuous $\forall\ \alpha$