
Defn: Let \( J \) be an index set. Given a set \( X \), a \textbf{J-tuple} of elements of \( X \) is a function \( x : J \rightarrow X \). The \textit{\( \alpha \)th coordinate} of \( x = x_{\alpha} = x(\alpha) \).

Defn: Let \( \{A_{\alpha}\}_{\alpha \in J} \) be an indexed family of sets. Let \( X = \bigcup_{\alpha \in J} A_{\alpha} \). The \textbf{Cartesian product} of \( \{A_{\alpha}\}_{\alpha \in J} \), denoted by \( \Pi_{\alpha \in J} A_{\alpha} \), is defined to be the set of all J-tuples \( (x_{\alpha})_{\alpha \in J} \) of elements of \( X \) such that \( x_{\alpha} \in A_{\alpha} \) for each \( \alpha \in J \).

That is, it is the set of all functions \( x : J \rightarrow \bigcup_{\alpha \in J} A_{\alpha} \) such that \( x(\alpha) \in A_{\alpha} \forall \alpha \in J \).

Defn: The \textbf{box topology} on \( \Pi_{\alpha \in J} X_{\alpha} \) is the topology generated by the basis
\[
\{ \Pi_{\alpha \in J} U_{\alpha} \mid U_{\alpha} \text{ open in } X_{\alpha} \}.
\]

Defn: Let \( S_{\alpha} = \{ \pi_{\alpha}^{-1}(U) \mid U \text{ open in } X_{\alpha} \} \) The \textbf{product topology} on \( \Pi_{\alpha \in J} X_{\alpha} \) is the topology generated by the subbasis \( S = \bigcup_{\alpha \in J} S_{\alpha} \).