Defn: $x \in X$ is a limit point of $A$ iff $x \in U^{\text{open}}$ implies $U \cap A - \{x\} \neq \emptyset$.

Defn: $A^\prime = \text{the set of all limit points of } A$.

Thm 17.6: $\overline{A} = A \cup A^\prime$.

Cor 17.7: $A$ closed if and only if $A^\prime \subset A$.

Defn: $x_n$ converges to a limit $x$ if for every neighborhood $U$ of $x$, there exists a positive integer $N$ such that $n \geq N$ implies $x_n \in U$.

Note: limit point of a set is not the same as limit of a sequence.

Defn: $X$ is Hausdorff space if for all $x_1, x_2 \in X$ such that $x_1 \neq x_2$, there exists neighborhoods $U_1$ and $U_2$ of $x_1$ and $x_2$, respectively, such that $U_1 \cap U_2 = \emptyset$.

Thm 17.8: Every finite point set in a Hausdorff space $X$ is closed.

Defn: $X$ is $T_1$ if every one point set is closed.

Thm 17.9: Let $X$ by $T_1$, $A \subset X$. Then $x$ is a limit point of $A$ if and only if every neighborhood of $x$ contains infinitely many points of $A$.

Thm 17.10: If $X$ is Hausdorff, then a sequence of points of $X$ converges to at most one point of $X$.

Thm 17.11: If $X$ has the order topology, then $X$ is Hausdorff. The product of two Hausdorff spaces is Hausdorff. A subspace of a Hausdorff space is Hausdorff.