Recall that a constant solution is an equilibrium solution. Thus its derivative is 0.

To find an equilibrium solution (i.e., constant solution), plug it in (for example, plug in $y(t) = k$ or $x_1(t) = k_1, x_2(t) = k_2$ depending on variables used and if you have one DE or a system of two DEs). Since the derivative of a constant is zero, this is equivalent to setting the derivative = 0.

Find all equilibrium solutions and classify them (stable, asymptotically stable, semi-stable, unstable and if system of DEs, node, saddle, spiral, center). In the case of non-linear system of DEs, state all possibilities for type of equilibrium solution.

If the (system of) differential equation(s) does not have an equilibrium solution, state so (note 4 of the following 16 problems below do not have an equilibrium solution).

Hint: The eigenvalues of upper and lower triangular matrices are the diagonal entries.
Note: You do not need to draw any direction fields.

1.) $y' = (y-3)^4(y-5)^9$ $\quad$ $y = 3$ is semi-stable, $y = 5$ is unstable.

2.) $y' = y^2 + 2$ $\quad$ no equilibrium solution.

3.) $y' = sin(y)$ $\quad$ $y = 2n\pi$ is unstable, $y = (2n+1)\pi$ is asymptotically stable.

4.) $y' = sin(t)$ $\quad$ no equilibrium solution.

5.) $y' = sin^2(y)$ $\quad$ $y = n\pi$ is semi-stable.

6.) $y' = sin^2(t)$ $\quad$ no equilibrium solution.

7.) $y' = ty$ $\quad$ $y = 0$ is unstable.

8.) $x' = 4 - y^2, \;\; y' = (x+1)(y-x)$

If $4 - y^2 = 0$, then $y = \pm 2$

If $y = 2$, then $(x+1)(y-x) = (x+1)(2-x) = 0$. Thus $x = -1, 2$.

If $y = -2$, then $(x+1)(y-x) = (x+1)(-2-x) = 0$. Thus $x = -1, -2$.

Jacobian matrix: $\begin{bmatrix} 0 & -2y \\ y - 2x - 1 & x + 1 \end{bmatrix}$

For $(x, y) = (-1, 2)$, Jacobian matrix is $\begin{bmatrix} 0 & -4 \\ 3 & 0 \end{bmatrix}$

Thus $(x(t), y(t)) = (-1, 2)$ is a stable center or unstable spiral or asymptotically stable spiral.

For $(x, y) = (2, 2)$, Jacobian matrix is $\begin{bmatrix} 0 & -4 \\ -3 & 3 \end{bmatrix}$

Thus $(x(t), y(t)) = (2, 2)$ is an unstable saddle.

For $(x, y) = (-1, -2)$, Jacobian matrix is $\begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix}$

Thus $(x(t), y(t)) = (-1, -2)$ is a stable center or unstable spiral or asymptotically stable spiral.

For $(x, y) = (-2, -2)$, Jacobian matrix is $\begin{bmatrix} 0 & 4 \\ 1 & -1 \end{bmatrix}$

Thus $(x(t), y(t)) = (-2, -2)$ is an unstable saddle.

9.) $x' = x - 2, \;\; y' = x - 1$ no equilibrium solution.

10.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \mathbf{x}$

One positive (1) and one negative eigenvalue (-2). Thus $(x_1(t), x_2(t)) = (0, 0)$ is an unstable saddle.

11.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} \mathbf{x}$

One positive (1) and one negative eigenvalue (-2). Thus $(x_1(t), x_2(t)) = (0, 0)$ is an unstable saddle.

12.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$

Purely imaginary eigenvalues $i, -i$. Thus $(x_1(t), x_2(t)) = (0, 0)$ is a stable center.

13.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} \mathbf{x}$

Two positive eigenvalues 1, 2. Thus $(x_1(t), x_2(t)) = (0, 0)$ is an unstable node.

14.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \mathbf{x}$

Two complex eigenvalues, $-1 \pm 2i$, with negative real part. Thus $(x_1(t), x_2(t)) = (0, 0)$ is an asymptotically stable spiral.

15.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -5 & 2 \end{bmatrix} \mathbf{x}$

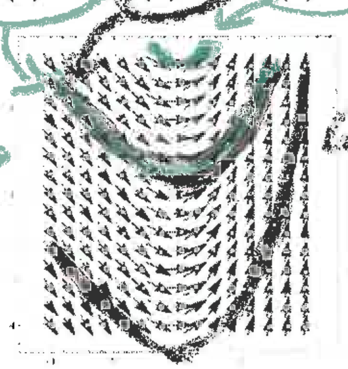
Two complex eigenvalues, $1 \pm 2i$, with positive real part. Thus $(x_1(t), x_2(t)) = (0, 0)$ is an unstable spiral.

16.) $\mathbf{x}' = \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix} \mathbf{x}$

Two negative eigenvalues -1, -2. Thus $(x_1(t), x_2(t)) = (0, 0)$ is an asymptotically stable node.

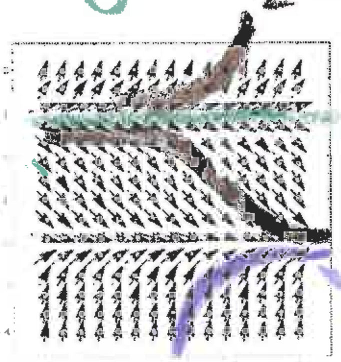
Problems 17 - 20 show the slope field for a first order differential equations. In addition to determining and classifying all equilibrium solutions (if any), also draw the trajectories satisfying the initial values $y(0) = 1$, $y(1) = 0$, $y(1) = 2$, $y(0) = -3$.

17.)



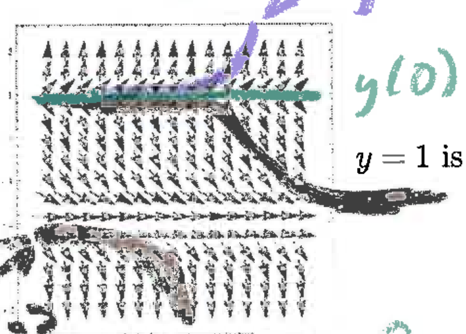
No equilibrium solution.

18.)



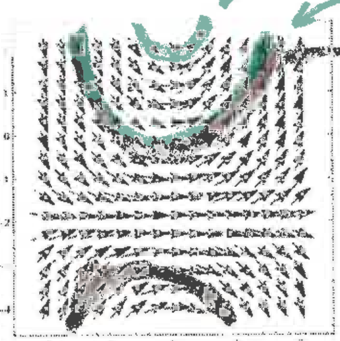
$y = 1$ is unstable. $y = -2$ is asymptotically stable.

19.)



$y = 1$ is unstable. $y = -2$ is semi-stable.

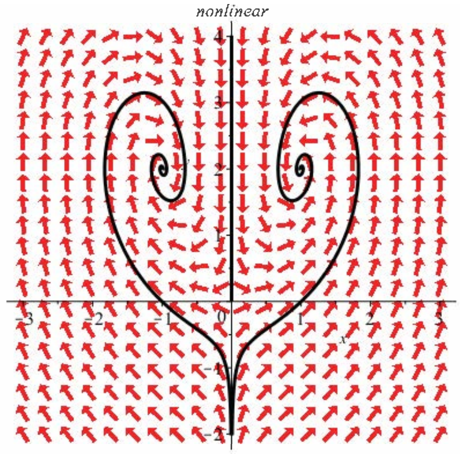
20.)



$y = -2$ is unstable.

Problems 21-23 show the stream plot in the $x_1 - x_2$-plane for a system of two first order differential equations In addition to determining and classifying all equilibrium solutions, also draw the trajectories satisfying the initial values $(x_1(0), x_2(0)) = (0, 1)$, $(x_1(0), x_2(0)) = (1, 0)$, $(x_1(0), x_2(0)) = (1, 2)$, $(x_1(0), x_2(0)) = (-1, 0)$. Also describe the basins of attraction.

21.)



$(x_1(t), x_2(t)) = (0, 0)$ is an unstable saddle.

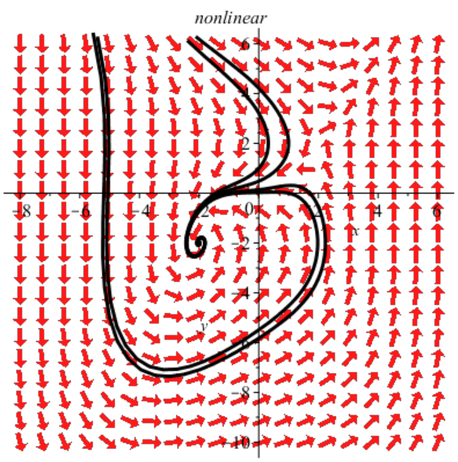
$(x_1(t), x_2(t)) = (1, 2)$ is an asymptotically stable node.

basin of attraction: $x_1 > 0$.

$(x_1(t), x_2(t)) = (-1, 2)$ is an asymptotically stable node.

basin of attraction: $x_1 < 0$.

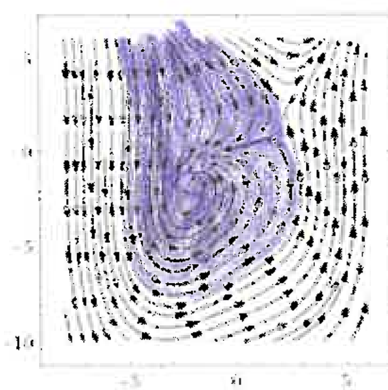
22.)



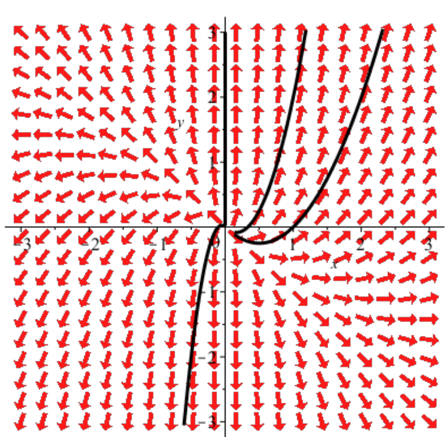
$(x_1(t), x_2(t)) = (2, 2)$ is an unstable saddle.

$(x_1(t), x_2(t)) = (-2, -2)$ is an asympt. stable spiral.

basin of attraction:



23.)



$(x_1(t), x_2(t)) = (0, 0)$ is an unstable node.

No basin of attraction: $x_1 < 0$.