Recall that a constant solution is an equilibrium solution. Thus its derivative is 0.

To find an equilibrium solution (i.e., constant solution), plug it in (for example, plug in $y(t) = k$ or $x_1(t) = k_1, x_2(t) = k_2$ depending on variables used and if you have one DE or a system of two DEs). Since the derivative of a constant is zero, this is equivalent to setting the derivative = 0.

Find all equilibrium solutions and classify them (stable, asymptotically stable, semi-stable, unstable and if system of DEs, node, saddle, spiral, center). In the case of non-linear system of DEs, state all possibilities for type of equilibrium solution.

If the (system of) differential equation(s) does not have an equilibrium solution, state so (note 4 of the following 16 problems below do not have an equilibrium solution).

Hint: The eigenvalues of upper and lower triangular matrices are the diagonal entries.
Note: You do not need to draw any direction fields.

1.) $y' = (y-3)^4(y-5)^9$

2.) $y' = y^2 + 2$

3.) $y' = sin(y)$

4.) $y' = sin(t)$

5.) $y' = sin^2(y)$

6.) $y' = sin^2(t)$

7.) $y' = ty$

8.) $x' = 4 - y^2, \;\; y' = (x+1)(y-x)$

9.) $x' = x - 2, \;\; y' = x - 1$

10.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \mathbf{x}$

11.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 5 & -2 \end{bmatrix} \mathbf{x}$

12.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$

13.) $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} \mathbf{x}$

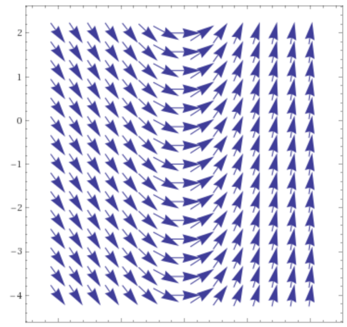
14.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \mathbf{x}$

15.) $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -5 & 2 \end{bmatrix} \mathbf{x}$

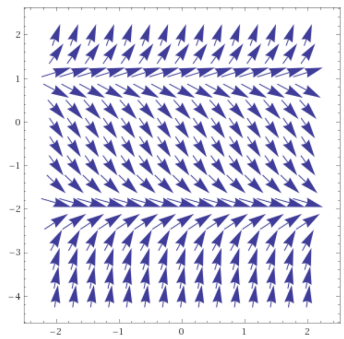
16.) $\mathbf{x}' = \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix} \mathbf{x}$

Problems 17 - 20 show the slope field for a first order differential equations. In addition to determining and classifying all equilibrium solutions (if any), also draw the trajectories satisfying the initial values $y(0) = 1$, $y(1) = 0$, $y(1) = 2$, $y(0) = -3$.

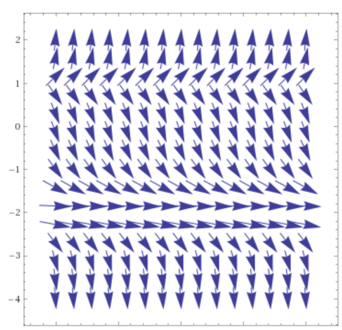
17.)



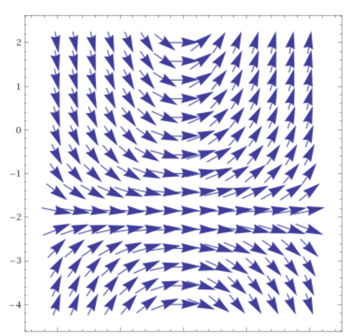
18.)



19.)

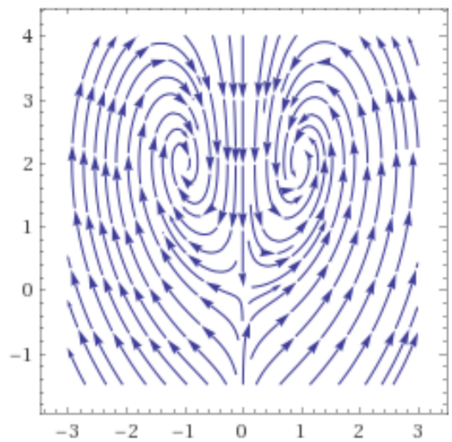


20.)

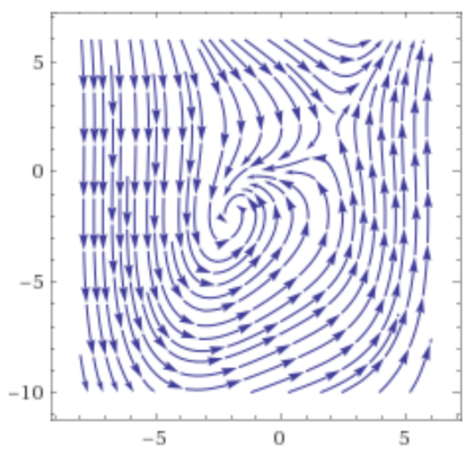


Problems 21-23 show the stream plot in the $x_1 - x_2$-plane for a system of two first order differential equations In addition to determining and classifying all equilibrium solutions, also draw the trajectories satisfying the initial values $(x_1(0), x_2(0)) = (0, 1)$, $(x_1(0), x_2(0)) = (1, 0)$, $(x_1(0), x_2(0)) = (1, 2)$, $(x_1(0), x_2(0)) = (-1, 0)$. Also describe the basins of attraction.

21.)



22.)



23.)