Suppose an object moves in the 2D plane (the x_1, x_2 plane) so that it is at the point $(x_1(t), x_2(t))$ at time t. Suppose the object's velocity is given by

$$x_1'(t) = ax_1 + bx_2,$$
$$x_2'(t) = cx_1 + dx_2$$

Or in matrix form $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

To solve, find eigenvalues and corresponding eigenvectors:

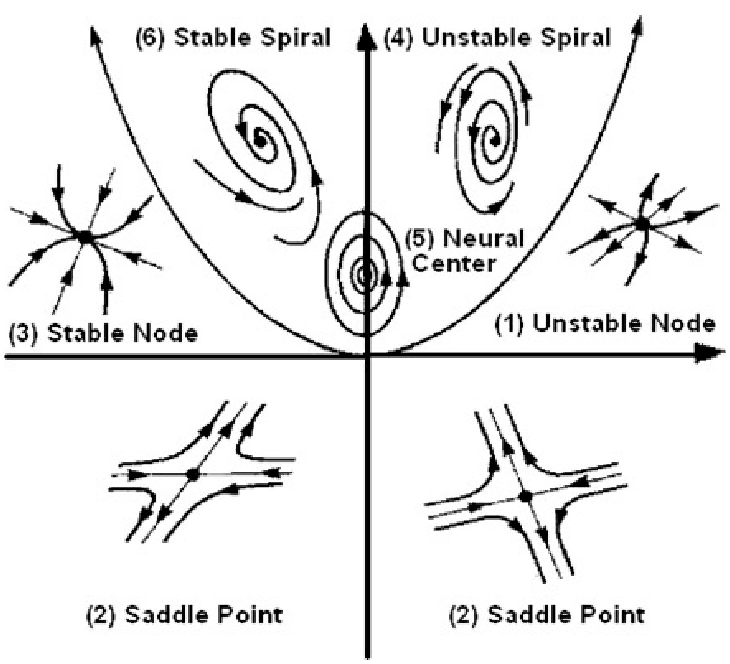
$$\begin{vmatrix} a-r & b \\ c & d-r \end{vmatrix} = (a-r)(d-r) - bc = r^2 - (a+d)r + ad - bc = 0.$$

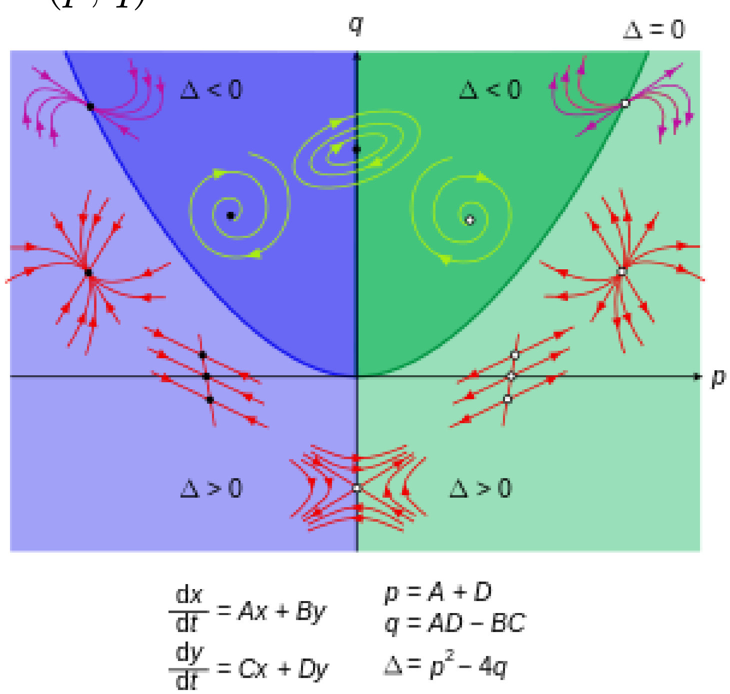
$$\text{Thus } r = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$$

Let $p = trace(A) = a + d$ and let $q = detA = ad - bc$

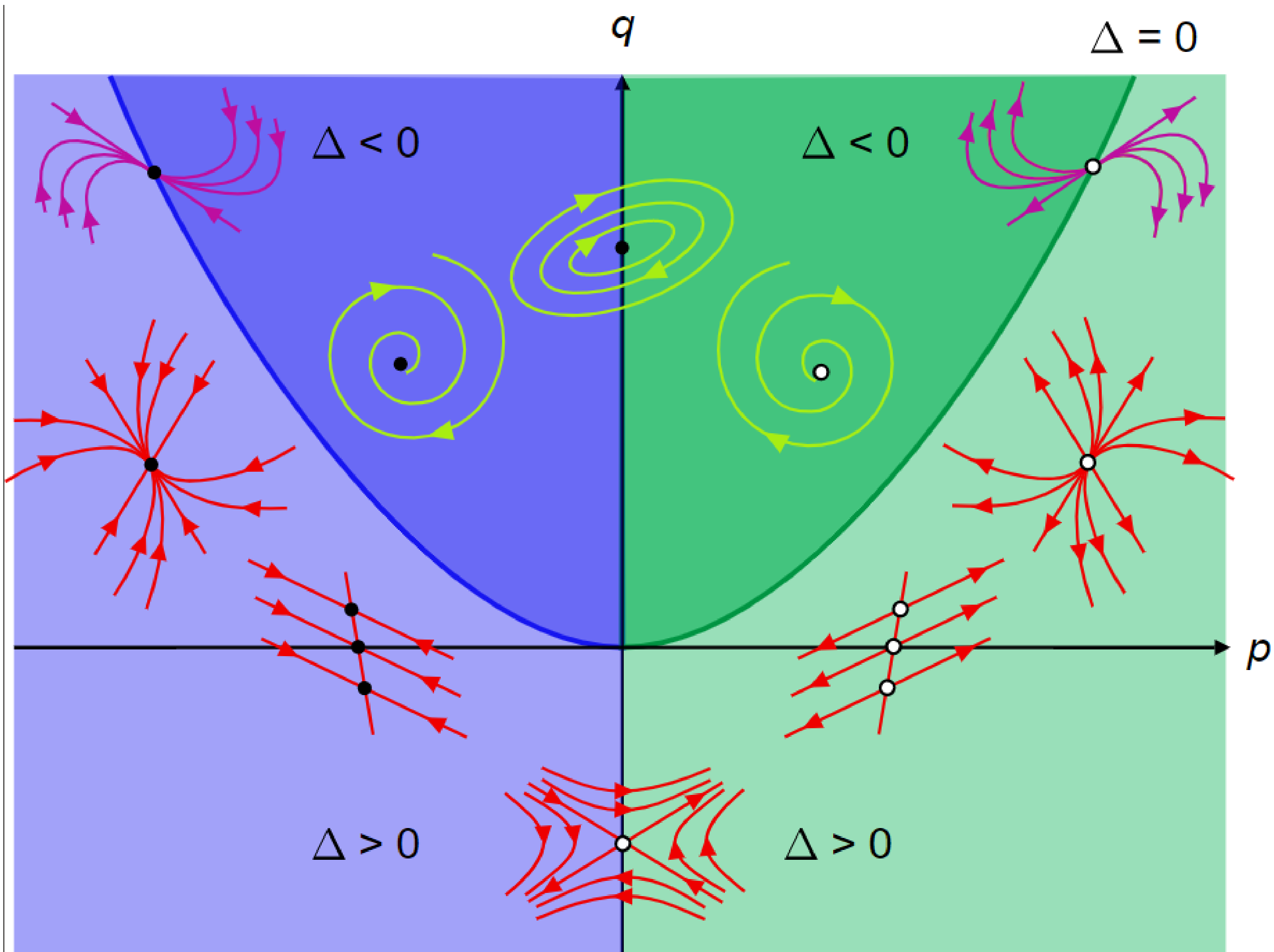
Then $r = \frac{p \pm \sqrt{p^2 - 4q}}{2}$

Thus the type of solution depends on (p, q)





http://digital-library.theiet.org/content/journals/10.1049/iet-gtd_20080456 https://en.wikipedia.org/wiki/Phase_plane



$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + By \qquad p = A + D$$

$$q = AD - BC$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = Cx + Dy \qquad \Delta = p^2 - 4q$$