$$\phi_4(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{3\cdot 2} + \frac{t^5}{5\cdot 3}$$

$$= \frac{2^0 t^2}{2} + \frac{2^1 t^3}{3\cdot 2} + \frac{2^2 t^4}{4\cdot 3\cdot 2} + \boxed{\frac{2^3 t^5}{5\cdot 4\cdot 3\cdot 2}}$$

$$\phi_4 = \sum_{K=2}^{5} \frac{2^{K-2}\, t^K}{K!} = \sum_{K=1}^{4} \frac{2^{K-1}\, t^{K+1}}{(K+1)!} = \sum_{K=0}^{3} \frac{2^K\, t^{K+2}}{(K+2)!}$$

started at $K=2$

$$\phi_n = \sum_{K=2}^{n+1} \frac{2^{K-2}\, t^K}{K!} = \sum_{K=1}^{n} \frac{2^{K-1}\, t^{K+1}}{(K+1)!}$$

started at $K=2$

$$= \sum_{K=0}^{n-1} \frac{2^K\, t^{K+2}}{(K+2)!}$$

3.1 (and 3.3, 3.4)

Solve 2^{nd} order homog LINEAR DE w/ constant coefficients

$$ay''+by'+cy=0$$

homogeneous

To solve, plug in educated guess:

$$y=e^{rt} \Rightarrow y'=re^{rt} \Rightarrow y''=r^2e^{rt}$$

$$ay''+by'+cy=0$$

$$ar^2\cancel{e^{rt}}+br\cancel{e^{rt}}+c\cancel{e^{rt}}=0/e^{rt}$$

$$\boxed{\underbrace{ar^2+br+c}_{\text{characteristic polynomial}}=0}$$

characteristic equation

$$ay'' + by' + cy = 0 \Rightarrow$$
$$ar^2 + br + c = 0$$

assumed $y = e^{rt}$

$$\Rightarrow r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $r_1 \neq r_2$ and r_1, r_2 are both real #'s

$\Rightarrow$ General sol'n

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$\phi_1 = e^{r_1 t}$ and $\phi_2 = e^{r_2 t}$ are both solns to $ay'' + by' + cy = 0$

Claim: $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$ is also a soln

In more generality if $y = \phi_1$ and $y = \phi_2$ are solns to $ay'' + by' + cy = 0$

$\Rightarrow y = c_1\phi_1 + c_2\phi_2$ is a soln

Proof: plug $y = c_1\phi_1 + c_2\phi_2$ into the LHS

$a(c_1\phi_1 + c_2\phi_2)'' + b(c_1\phi_1 + c_2\phi_2)' + c(c_1\phi_1 + c_2\phi_2)$

$= ac_1\phi_1'' + ac_2\phi_2'' + bc_1\phi_1' + bc_2\phi_2' + cc_1\phi_1 + cc_2\phi_2$

$= c_1(a\phi_1'' + b\phi_1' + c\phi_1) + c_2(a\phi_2'' + b\phi_2' + c\phi_2)$

$= c_1(0) + c_2(0) \overset{\checkmark}{=} 0$ $\square$