MATH2560 Eng. Math. IV Differential Equations

Quiz 4 Name________________________ Section__________

[8] 1. A 2 kg object attached to a spring will stretch the spring 980 cm. The mass is also attached to a viscous damper that exerts a force of 10N when the velocity of the mass is 2m/sec. No external force is applied to the object. The object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward. State the 2nd order initial value problem that models the motion of the mass. Note $g = 9.8$ **meters**/sec^2. **Do NOT solve.**

Differential equation: ____

Initial values: ____

[6] 2. Solve $y''' - 3y'' = 0$

Solution: ____

[6] 3. Solve $y''' + 2y'' - y' - 2y = 0$

Solution: ____

Answers:

[8] 1. A 2 kg object attached to a spring will stretch the spring 980 cm. The mass is also attached to a viscous damper that exerts a force of 10N when the velocity of the mass is 2m/sec. No external force is applied to the object. The object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward. State the 2nd order initial value problem that models the motion of the mass. Note $g = 9.8$ **meters**/sec^2. **Do NOT solve.**

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A 2 kg object attached to a spring will stretch the spring 980 cm $\Rightarrow m = 2, L = 9.8m$, $kL = mg$ implies $9.8k = (2)(9.8)$. Thus $k = 2$.

The mass is also attached to a viscous damper that exerts a force of 10N when the velocity of the mass is 2m/sec and $|F_{damping}| = |\gamma|v \Rightarrow 10 = \gamma(2)$, and thus $\gamma = 5$.

No external force is applied to the object $\Rightarrow F_{external} = 0$

The object is initially displaced 20 cm downward from its equilibrium position $\Rightarrow u(0) = +0.2m$.

and given a velocity of 10 cm/sec upward $\Rightarrow u(0) = -0.1m$

Differential equation: $\underline{\quad 2u'' + 5u' + 2u = 0 \quad}$

Initial values: $\underline{\quad \text{u}(0) = 0.2, \quad u(0) = -0.1 \quad}$

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[6] 2. Solve $y''' - 3y'' = 0$

$r^3 - 3r^2 = 0$ implies $r^2(r-3) = 0$. Thus $r = 0, 0, 3$

Hence 3 linearly independent solutions are $y = e^{0t} = 1, \;\; y = te^{0t} = t$, $y = e^{3t}$

Solution: $\underline{\quad y = c_1 + c_2 t + c_3 e^{3t} \quad}$

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[6] 3. Solve $y''' + 2y'' - y' - 2y = 0$

$r^3 + 2r^2 - r - 2 = 0$

$r^2(r+2) - (r+2) = (r^2 - 1)(r+2) = 0$. Thus $r = \pm 1, -2$

Alternative method if you don't notice $r + 2$ is a factor: Find a root, for example, $r = 1$ is a solution. Hence

$r^3 + 2r^2 - r - 2 = (r-1)(r^2 + xr + 2) = r^3 + (x-1)r^2 + (2-x)r + 2$. Thus $x - 1 = 2$ and $x = 3$

$r^3 + 2r^2 - r - 2 = (r-1)(r^2 + 3r + 2) = (r-1)(r+1)(r+2) = 0$. Thus $r = \pm 1, -2$.

Solution: $\underline{\quad y = c_1 e^t + c_2 e^{-t} + c_3 e^{-2t} \quad}$