Exam 2: 6:30PM - 8:00PM Monday April 11 in LR1 VAN over Chapters 3 and 4 and 6.1-6.4 (but NOT 4.4).

on ICON via announcement

To study for exam 2:

1.) If you can do all the review problems, you should be in good shape.

2.) If you have difficulty with factoring, check out Solving higher order linear homogeneous DE and the answers to quiz 4 (and Friday's class notes).

3.) If you have difficulty with the LaPlace transform, check out Intro to Ch 6: LaPlace Transform. Note to perform the inverse transform requires only a few specific operations as described on page 2 of table of LaPlace transforms.

NOTE the (shorter) 6.2 problems are at the end of HW 9. Please make sure you do those before exam 2 even though they are at the end of the assignment. Since the 6.2 problems are shorter, you may want to do them first.

I forgot that 6.5 is covered on this HW. Since it is not on the exam, I have postponed the due date of HW 9 to THURSDAY (not Sunday), so you don't need to do the 6.5 HW problems until after Monday's exam 2. But if you would like extra practice, you can also do the 6.5 problems earlier.

4.) To see various different wordings for mechanical vibration problems, see Mechanical Vibration review.

R $\frac{2\pi}{\omega}$ ω δ

To determine amplitude, period, frequency, and phase, convert to Rcos(wt - d).

displacement

Note (R, d) can be found by converting (c1, c2) to polar coordinates per Some review problems including answers. These old quiz problems with answers are also good review problems.

See MV slides for a review of damping (critical, under, over) as well as resonance, etc.

Note polar coordinates can be used to factor some polynomials. See class notes.

To pass this class, you don't need to know everything. Concentrate on the basics (and your favorites) if you are short of time.

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$R \quad \frac{2\pi}{\omega} \quad \omega \quad \delta$

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external force freq matching homogeneous freq

To pass this class, you don't need to know everything. Concentrate on the basics (and your favorites) if you are short of time.

No damping $\Rightarrow$ resonance occurs at $\sqrt{\frac{k}{m}}$

$mu'' + ku = 0$

$mr^2 + k = 0 \Rightarrow r = \pm i\sqrt{\frac{k}{m}}$

If small damping (underdamped) resonance occurs near $\sqrt{\frac{k}{m}}$

Last year I created a discussion page asking students to create short video lectures per below. We won't ask you to create videos this semester, but students did much much better than usual on chapter 6 exam problems last year. Thus I am copying some of the discussion to this page for an introduction/review of LaPlace transforms. Please ignore comments regarding videos.

In chapter 6 we will use the LaPlace transform to turn a linear DE into an algebra problem using linearity and a table of LaPlace transforms.

For example to solve

$$\mathcal{L}(y'' + 3y' + 4y = 0), \quad y(0) = 5, \quad y'(0) = 6$$

1.) Take the LaPlace Transform of both sides of the DE equation:

$$\mathcal{L}(y'' + 3y' + 4y) = \mathcal{L}(0)$$

or use table for $\mathcal{L}(g(t))$

2.) Use the fact that the LaPlace Transform is linear:

$$\mathcal{L}(y'') + 3\mathcal{L}(y') + 4\mathcal{L}(y) = 0$$

3.) Use formula 18 to change this equation into an algebraic equation:

$$[s^2\mathcal{L}(y) - sy(0) - y'(0)] + 3[s\mathcal{L}(y) - y(0)] + 4\mathcal{L}(y) = 0$$

3.5) Substitute in the initial values:

$$s^2\mathcal{L}(y) - 5s - 6 + 3[s\mathcal{L}(y) - 5] + 4\mathcal{L}(y) = 0$$

4.) Solve the algebraic equation for $\mathcal{L}(y)$:

$$s^2\mathcal{L}(y) - 5s - 6 + 3s\mathcal{L}(y) - 15 + 4\mathcal{L}(y) = 0$$

$$[s^2 + 3s + 4]\mathcal{L}(y) = 5s + 21$$

$$\mathcal{L}(y) = \frac{5s+21}{s^2+3s+4}$$

Some algebra implies $\boxed{\mathcal{L}(y) = \frac{5s+21}{s^2+3s+4}}$

5.) Solve for y by taking the inverse LaPlace transform of both sides:

$\mathcal{L}(y') = s(\underline{\mathcal{L}(y)}) - \underline{y(0)}$

$\mathcal{L}(y'') = s^2(\underline{\mathcal{L}(y)}) - s(\underline{y(0)}) - \underline{y'(0)}$

$\mathcal{L}(y''') = s^3(\underline{\mathcal{L}(y)}) - s^2(\underline{y(0)}) - s(\underline{y'(0)}) - \underline{y''(0)}$

$\mathcal{L}(y'''') = s^4(\underline{\mathcal{L}(y)}) - s^3(\underline{y(0)}) - s^2(\underline{y'(0)}) - s(\underline{y''(0)}) - \underline{y'''(0)}$

stop at constant

Note the pattern. Lets now fill in the first blank for each of these. Note that the coefficient of s^n in Formula 18 is $\mathcal{L}(y)$.

Thus if we fill in just the first blank in each of the above, we get

$\mathcal{L}(y') = s\mathcal{L}(y) - \underline{y(0)}$

$\mathcal{L}(y'') = s^2\mathcal{L}(y) - s(\underline{y(0)}) - \underline{y'(0)}$

$\mathcal{L}(y''') = s^3\mathcal{L}(y) - s^2(\underline{y(0)}) - s(\underline{y'(0)}) - \underline{y''(0)}$

$\mathcal{L}(y'''') = s^4\mathcal{L}(y) - s^3(\underline{y(0)}) - s^2(\underline{y'(0)}) - s(\underline{y''(0)}) - \underline{y'''(0)}$

The remaining blanks are filled in with initial values, starting with y(0). As the degree of s decreases, the derivative of y (evaluated at 0) increases. Note we always end with subtracting a constant term as we run out of initial values at the same time we run out of s's.

Thus our formulas become:

$$\mathcal{L}(y') = s\mathcal{L}(y) - y(0)$$

$$\mathcal{L}(y'') = s^2\mathcal{L}(y) - sy(0) - y'(0)$$

$$\mathcal{L}(y''') = s^3\mathcal{L}(y) - s^2y(0) - sy'(0) - y''(0)$$

$$\mathcal{L}(y'''') = s^4\mathcal{L}(y) - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0)$$

R = amplitude

Quiz 3 Section 93 Oct 18, 2019

$R\cos(\omega t - \delta)$ | ω = freq, δ = phase = displacement

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

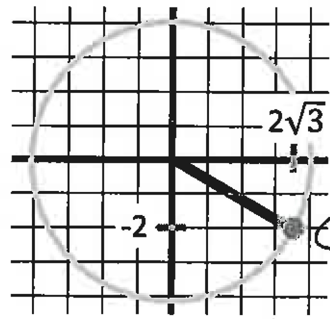
period $\frac{2\pi}{2}$, freq = 2

$u(t) = 2\sqrt{3}cos(2t) - 2sin(2t) =$ ______________________

$c_1\cos(2t) + c_2\sin(2t)$

$= R\cos\delta\cos(2t) + R\sin\delta\sin(2t)$

$= R\cos(2t - \delta)$



$(c_1, c_2) = (R\cos\delta, R\sin\delta)$

(4) 3.6 [3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

variation of parameter

$y(t) = u_1(t)\phi_1(t) + u_2(t)\phi_2(t)$

To solve for u_1 & u_2 $\Rightarrow$ $\underbrace{\begin{bmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{bmatrix}}_{\text{Wronskian}} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ g(t)/a \end{bmatrix}$

where $u_1(t) =$ ______________________ and $u_2(t) =$ ______________________

[4] 3.) A spring mass system has a spring constant of 3N/m. A mass of 4kg is attached to the spring. The system is driven by an external force of sin(3t) N. The spring is streched 5 m and then set in motion with a downward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: ____

Choose $u_1'\phi_1 + u_2'\phi_2 = 0$

Plug in & got $u_1'\phi_1' + u_2'\phi_2 = g(t)/a$

$W(\phi_1, \phi_2)\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ g(t)/a \end{bmatrix}$

[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = u_1(t)\phi_1 + u_2(t)\phi_2$$

where $u_1(t) = \int \frac{\begin{vmatrix} 0 & \phi_2 \\ 1 & \phi_2' \end{vmatrix}}{\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix}} \frac{g(t)}{a} dt$ and $u_2(t) = \int \frac{\begin{vmatrix} \phi_1 & 0 \\ \phi_1' & 1 \end{vmatrix}}{\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix}} \frac{g(t)}{a} dt$

solved for u_1 & u_2 using Cramer's rule & then integrate

[4] 3.) A 10 kg mass stretches a spring 5 m. The mass is acted on by an external force of 6cos(3t) N. The spring is compressed 4 m and then set in motion with an upward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: $10u'' + 19.6u = 6cos(3t), \; u(0) = -4, u'(0) = -2$

$mg = kL, \; k = \frac{10(9.8)}{5} = 2(9.8) = 19.6$

[4] 3.) A 20 kg mass stretches a spring 2 m. The mass is acted on by an external force of 9sin(5t) N and moves in a medium that imparts of viscous force of 8N when the speed of the mass is 4m/sec. The spring is compressed 4 m and then released. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: $20u'' + 2u' + 98u = 9sin(5t), \; u(0) = -4, u'(0) = 0$

$mg = kL, \; k = \frac{20(9.8)}{2} = 98$

$8 = \gamma(4)$. Thus $\gamma = 2$

[4] 3.) A spring mass system has a spring constant of 3N/m. A mass of 4kg is attached to the spring. The system is driven by an external force of sin(3t) N. The spring is streched 5 m and then set in motion with a downward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: $4u'' + 3u = sin(3t), \; u(0) = 5, u'(0) = 2$

[4] 3.) A spring is stretched 1m by a force of 2N. A mass of 9kg is attached to the spring and also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 1m/sec. The spring is set in motion from its equilibrium position with an upward velocity of 2m/s. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: $9u'' + 3u' + 2u = 0, \; u(0) = 0, u'(0) = -2$

$2 = 1k$. Thus $k = 2$

$3 = \gamma(1)$.

Quiz 3 Section 93 Oct 18, 2019

$b = \omega = 2 =$ frequency

period $= \frac{2\pi}{2}$

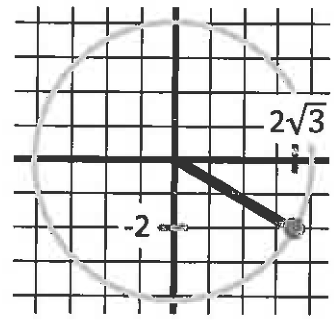
[3] 1.) Write the following in the form $u(t) = Rcos(\omega t - \delta)$

$u(t) = 2\sqrt{3}cos(2t) - 2sin(2t) =$ ______________________________

$= c_1 \cos(2t) + c_2 \sin(2t)$

$= R\cos(\delta)\cos(2t) + R\sin(\delta)\sin(2t)$

$= R\cos(2t - \delta)$



[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$y(t) =$ ____

where $u_1(t) =$ ________________________ and $u_2(t) =$ ________________________

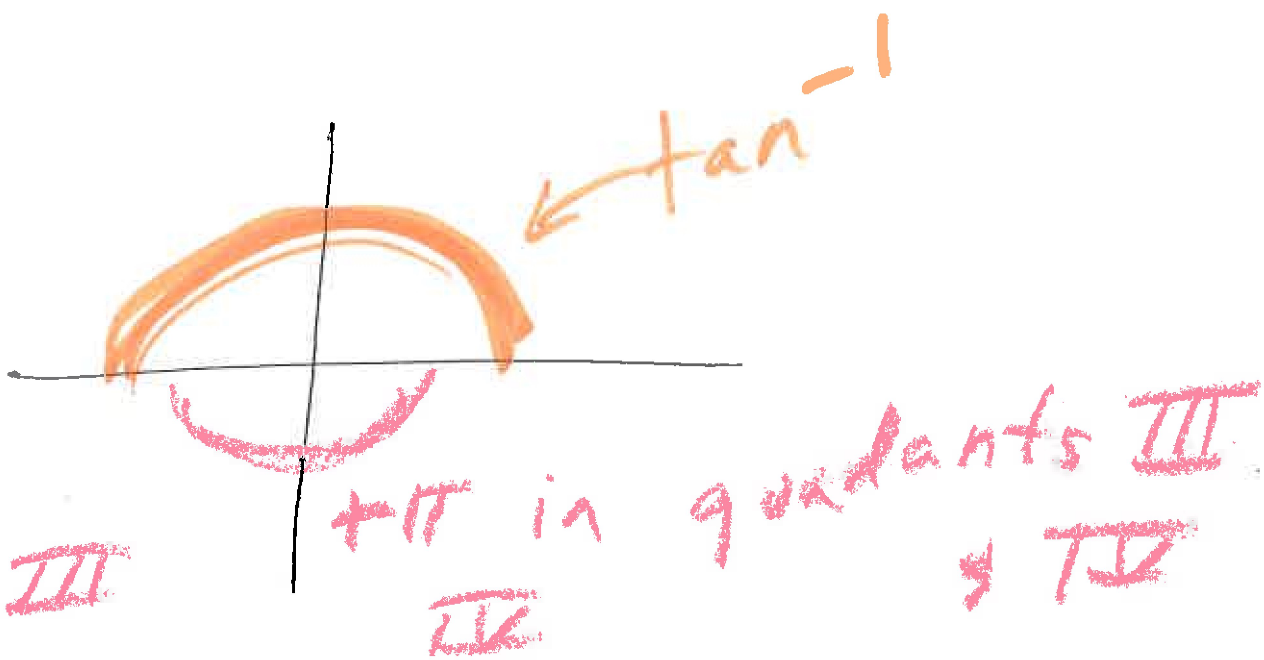
[4] 3.) A spring mass system has a spring constant of 3N/m. A mass of 4kg is attached to the spring. The system is driven by an external force of sin(3t) N. The spring is streched 5 m and then set in motion with a downward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve.**

IVP: ____

$$\left.\begin{aligned} c_1 &= R\cos\delta \\ c_2 &= R\sin\delta \end{aligned}\right\} \quad R = \sqrt{c_1^2 + c_2^2}$$

$$\tan\delta = \frac{R\sin\delta}{R\cos\delta} = \frac{c_2}{c_1}$$

$$\Rightarrow \delta = \tan^{-1}\left(\frac{c_2}{c_1}\right) + \underbrace{\text{maybe } \pi}_{}$$



Quiz 3 Section 91 Oct 18, 2019 $R = 4 = \sqrt{c_1^2 + c_2^2} = \sqrt{(-2)^2 + 2(\sqrt{3})^2}$

no $+\pi$

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$u(t) = -2cos(2t) + 2\sqrt{3}sin(2t) = \underline{4cos(2t - \frac{2\pi}{3})}$

$= c_1 \cos(2t) + c_2 \sin(2t)$

$= R\cos(\delta) \cos(2t) + R\sin\delta \sin(2t)$

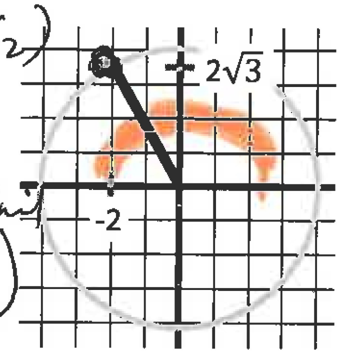
$= R\cos(2t - \delta)$

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

(c_1, c_2)

polar coordinates

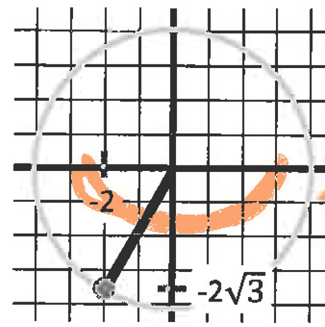
(R, δ)



[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$u(t) = -2cos(2t) - 2\sqrt{3}sin(2t) = \underline{4cos(2t + \frac{2\pi}{3})}$

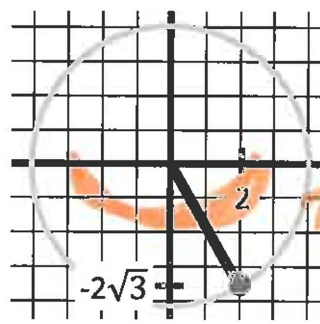
$-\frac{\pi}{2} - \frac{\pi}{6} = -\frac{4\pi}{6} = -\frac{2\pi}{3}$



$+\pi$

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$u(t) = 2cos(2t) - 2\sqrt{3}sin(2t) = \underline{4cos(2t + \frac{\pi}{3})}$



$+\pi$

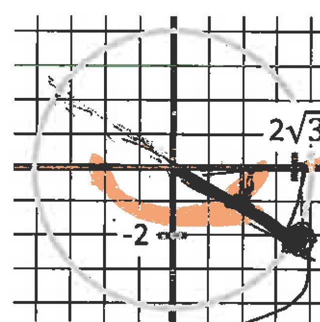
[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$u(t) = 2\sqrt{3}cos(2t) - 2sin(2t) = \underline{4cos(2t + \frac{\pi}{6})}$

$c_1 = R\cos\delta = 2\sqrt{3}$

$c_2 = R\sin\delta = -2$

$R = 4 = \sqrt{(2\sqrt{3})^2 + (-2)^2}$



$+\pi$

(c_1, c_2) or in polar coord (R, δ)

$\boxed{\delta = \frac{-\pi}{6}} \leftarrow \tan\delta = \frac{R\sin\delta}{R\cos\delta} = \frac{c_2}{c_1} = \frac{-2}{2\sqrt{3}}$

$\delta = \tan^{-1}\left(\frac{c_2}{c_1}\right) + \text{maybe } \pi$