2.1 Solve 1rst order linear DE

$$1\,y' + p(t)\,y = g(t)$$

yy' not linear
$ay + by'$

use integrating factor

$$u(t) = e^{\int p(t)\,dt}$$

Ex: $1y' + 3t^2 y = t^2$

$p(t) = 3t^2$

Find integrating factor $u(t) = e^{\int p(t)dt} = e^{\int 3t^2 dt} = e^{t^3+C}$

need only one $u(t) = e^{t^3}$

Multiply both sides by integrating factor

$$e^{t^3}\left[y' + 3t^2 y\right] = t^2 e^{t^3}$$

LHS

$$e^{t^3} y' + 3t^2 e^{t^3} y = t^2 e^{t^3}$$

check product rule

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$$(e^{t^3} \cdot y)' = t^2 e^{t^3}$$

$u(t)$

$$\int (e^{t^3} y)'\,dt = \int t^2 e^{t^3}\,dt$$

FTC

LHS

$u(t)$

$$e^{t^3} \cdot y = \int t^2 e^{t^3}\,dt$$ RHS

Let $v = t^3$

$\frac{1}{3} dv = \frac{3t^2}{3} dt$

To integrate RHS

- integration by substition

To integrate RHS
- integration by substitution
- " " parts
- " " partial fractions

$\frac{1}{3}dv = \frac{3t}{3}$ a.

$= \frac{1}{3}\int e^{v}\,dv$

$= \frac{1}{3}e^{v} + C$

$\frac{e^{t^3}}{\cancel{e^{t^3}}} \cdot y = \frac{1}{3}\frac{e^{t^3}}{\cancel{e^{t^3}}} + \frac{C}{e^{t^3}}$

$$\boxed{y = \frac{1}{3} + Ce^{-t^3}}$$

Ex 2: $t^3 y' + 3t^2 y = 4$ notice product rule

$\div t^3$

$$\boxed{\int (t^3 y)'\,dt = \int 4\,dt}$$

$1\,y' + \frac{3}{t}y = 4t^{-3}$

$u(t) = e^{\int p(t)} = e^{\int \frac{3}{t}dt}$

$= e^{3\ln|t|} = \cancel{e}^{\cancel{\ln}|t|^3} = |t|^3$

Let $u(t) = t^3$

$t^3\left(y' + \frac{3}{t}y\right) = \left(4t^{-3}\right)t^3$

$$\boxed{t^3 y' + 3t^2 y = 4}$$

$$\boxed{(t^3 y)' = 4}$$

check product rule

$\underbrace{\int (t^3 y)'\,dt}_{\downarrow \text{FTC}} = \int 4\,dt$

FTC:

$\int F'(t)\,dt = F(t) + C$

$\downarrow$ FTC

$$t^3 \cdot y = 4t + C$$

$u(t)\cdot y$

Solve for y:

$$\boxed{y = 4t^{-2} + Ct^{-3}}$$

Solve IVP $\underbrace{t^3 y' + 3t^2 y = 4}_{DE}$, $\underbrace{y(1) = 10}_{\text{initial value}}$

solve DE: $y = 4t^{-2} + Ct^{-3}$

plug in initial value to find C

$$10 = 4(1)^{-2} + C(1)^{-3}$$

$$10 = 4 + C \Rightarrow C = 6$$

$$\boxed{\text{IVP soln: } y = 4t^{-2} + 6t^{-3}}$$

Note currently you only have 2 techniques for solving DE

Thus to solve, you need to determine if DE is

1. Linear: $1y' + p(t)y = g(t)$
2. separable: separate variables
3. Both

ex: $2y' + 3y = 4$ is both linear & separable

In section 2.4 we will learn a 3rd for a special case (Bernoulli eqn)