

1.1 : Falling Ball example



Force due to air resistance

$F_{\text{air resistance}}$ proportional to velocity

$$F_{\text{air resistance}} = \gamma v$$

$v = \text{velocity}$

(+)
 positive direction points down

Force acting on ball

$$= F_{\text{gravitational}} + F_{\text{air resistance}}$$

$$= mg - \gamma v$$

$$= ma = m \frac{dv}{dt}$$

Newton's
2nd law
 $F = ma$

Diff eqn: $m \frac{dv}{dt} = mg - \gamma v$

Throw the ball up

↑ movement is up

↑ movement is -

$$F_{\text{gravity}} = mg$$



$$F_{\text{air resistance}} = \gamma v$$

↑ acts in opposite of direction of motion

positive direction points down (+)

$$F = F_{\text{gravity}} + F_{\text{air resistance}} = mg - \gamma v$$

- γv needs to ~~should~~ be positive ^{but it isn't} but v is negative since ball is going up in the negative direction

Thus $F_{\text{air resistance}} = -\gamma v$

$$v = \frac{dx}{dt} \approx \frac{x_2 - x_1}{t_2 - t_1} < 0 \quad \uparrow$$

If ball is moving up or down

$$m \frac{dv}{dt} = mg - \gamma v$$

Suppose $m = 10 \text{ kg}$ $\gamma = 2$

$$\frac{10}{10} \frac{dv}{dt} = \frac{10(9.8)}{10} - \frac{2}{10} v$$

10

10

10

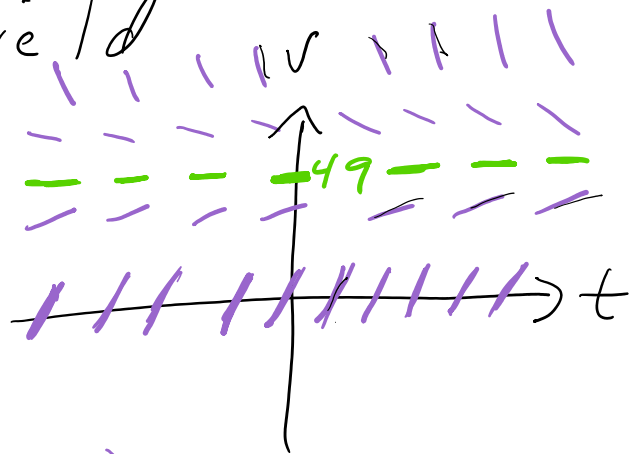
$$\frac{dv}{dt} = 9.8 - \frac{1}{5}v$$

to analyze velocity vs time

① solve differential eqn
- will do this on Wednesday

② Draw direction field
= drawing small portions of tangent lines in the t-v plane

= slope field



t	v	slope = $\frac{dv}{dt} = 9.8 - \frac{1}{5}v$
	0	$\frac{dv}{dt} = 9.8$
	49	0
	40	small positive #
	50	small negative #
	60	larger negative (steeper)

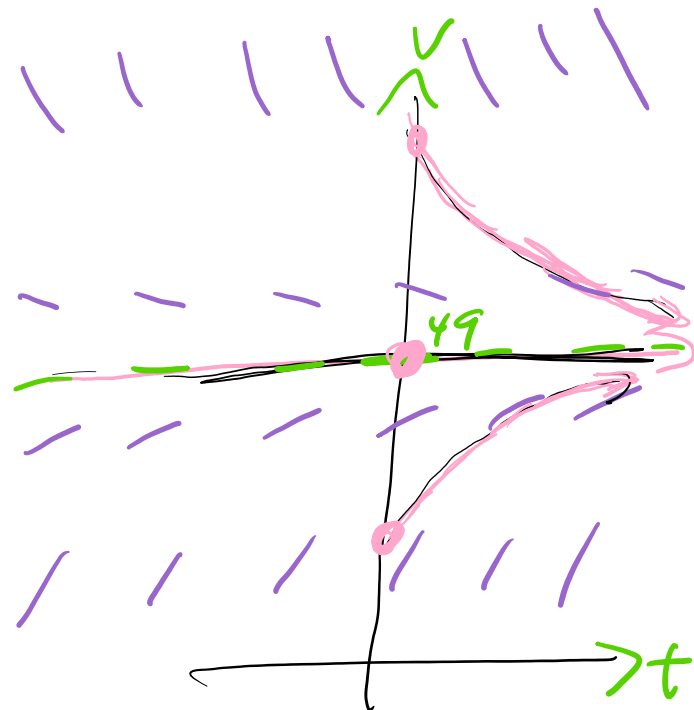
slope does not depend on t

slope 0: $0 = 9.8 - \frac{1}{5}v$

slope 0: $0 = 9.8 - \frac{1}{5}v$
 $v = (9.8)5 = 49$

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

\uparrow \ominus
 49 slope 0
 \downarrow \oplus



As $t \rightarrow \infty$, $v \rightarrow 49 \text{ m/sec}$

Equilibrium solutions
 = constant solns

ie $v = \text{constant}$



equil soln
 $v = 49$

$$\frac{dv}{dt} = \bigcirc$$



Previous Knowledge required for calculus

★ Integration by substitution,
parts, ★ partial fractions ★

★ tangent lines
including differentiation

• FTC (will quickly review)