Sect.3.6: **Guess $y = u_1(t)e^t + u_2(t)te^t$ and solve for u_1 and u_2**

$y' = u_1'e^t + u_1e^t + u_2'te^t + u_2(e^t + te^t) = e^{2t} + te^{2t} - te^{2t} - e^{2t}$.

Two unknown functions, u_1 and u_2, but only one equation ($y'' - 2y' + y = e^t ln(t)$). Thus might be OK to choose 2nd eq'n.

Avoid 2nd derivative in y'': $\boxed{\textbf{Choose } u_1'e^t + u_2'te^t = 0}$

Hence $y' = u_1e^t + u_2(e^t + te^t)$.

and $y'' = u_1'e^t + u_1e^t + u_2'(e^t + te^t) + u_2(e^t + e^t + te^t)$.
$= u_1'e^t + u_1e^t + u_2'e^t + u_2'te^t + u_2(2e^t + te^t)$.
$= u_1e^t + u_2'e^t + u_2(2e^t + te^t)$. ← no u_1'' or u_2''

Solve $y'' - 2y' + y = e^t ln(t)$

$$\boxed{u_1e^t + u_2'e^t + u_2(2e^t + te^t) - 2[u_1e^t + u_2(e^t + te^t)] + u_1e^t + u_2te^t = e^t ln(t)}$$

$u_2'e^t + 2u_2e^t + u_2te^t - 2u_2e^t - 2u_2te^t + u_2te^t = e^t ln(t)$

$u_2' = ln(t)$ or in other words, $\frac{du_2}{dt} = ln(t)$

Thus $\int du_2 = \int ln(t)dt$

$u_2 = tln(t) - t$. Note only need one solution, so don't need $+C$.

$y = u_1(t)e^t + [tln(t) - t]te^t$

$u_1'e^t + u_2'te^t = 0$. Thus $u_1' + u_2't = 0$. Hence $u_1' = -u_2't = -tln(t)$

Thus $u_1 = -\int tln(t)dt = -\frac{t^2 ln(t)}{2} + \frac{t^2}{4}$

Thus the general solution is

$$y = c_1e^t + c_2te^t + \left(-\frac{t^2 ln(t)}{2} + \frac{t^2}{4}\right)e^t + (tln(t) - t)te^t$$