

$$X' = \begin{pmatrix} 2 & 0 \\ 4 & 5 \end{pmatrix} X$$

$$\text{Find eigenvalues} \Rightarrow \begin{vmatrix} 2-\lambda & 0 \\ 4 & 5-\lambda \end{vmatrix} = (2-\lambda)(5-\lambda)$$

\Rightarrow eigenvalues are 2 and 5

$$\lambda = 2 \quad \begin{pmatrix} 2-2 & 0 \\ 4 & 5-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 4 & 3 \end{pmatrix} \Rightarrow X_1 = -\frac{3}{4}X_2$$

\Rightarrow eigenvector is $\begin{pmatrix} -3/4 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$$\lambda = 5 \quad \begin{pmatrix} 2-5 & 0 \\ 4 & 5-5 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 4 & 0 \end{pmatrix} \Rightarrow X_1 = 0 \quad X_2 \text{ free}$$

\Rightarrow eigenvector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

\Rightarrow general solution is

$$X(t) = C_1 e^{2t} \begin{bmatrix} -3 \\ 4 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$