

## Quiz #6 Solutions.

$$\#1 \quad \mathcal{L}^{-1}\left(\frac{2}{s^2+10s+34}\right) = \mathcal{L}^{-1}\left(\frac{2}{(s+5)^2+9}\right) = \mathcal{L}^{-1}\left(\frac{2}{3} \cdot \frac{3}{(s+5)^2+9}\right)$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{2}{s^2+10s+34}\right) = \frac{2}{3} e^{-5t} \sin(3t) \quad a=-5 \quad b=3$$

$$\#2. \quad X' = \begin{pmatrix} 2 & 0 \\ 4 & 5 \end{pmatrix} X$$

$$\text{Find eigenvalues} \Rightarrow \begin{vmatrix} 2-\lambda & 0 \\ 4 & 5-\lambda \end{vmatrix} = (2-\lambda)(5-\lambda)$$

$\Rightarrow$  eigenvalues are 2 and 5

$$\lambda=2 \quad \begin{pmatrix} 2-2 & 0 \\ 4 & 5-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 4 & 3 \end{pmatrix} \Rightarrow X_1 = -\frac{3}{4}X_2$$

$$\Rightarrow \text{eigenvector is } \begin{pmatrix} -3/4 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\lambda=5 \quad \begin{pmatrix} 2-5 & 0 \\ 4 & 5-5 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 4 & 0 \end{pmatrix} \Rightarrow X_1=0 \quad X_2 \text{ free}$$

$$\Rightarrow \text{eigenvector is } \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\Rightarrow$  general solution is

$$X(t) = C_1 e^{2t} \begin{bmatrix} -3 \\ 4 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$