$$mr'' = \frac{-GMm}{r^2}$$

Let $v = r'$, then $v' = r''$

Thus we obtain system of non-linear equations:

$$r' = v$$
$$v' = \frac{-GM}{r^2}$$

Note $v' = \frac{-GM}{r^2}$ involves 3 variables: v, t, r

Eliminate t: $v' = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr}v$

Thus $mv' = \frac{-GMm}{r^2}$ becomes $m\frac{dv}{dr}v = \frac{-GMm}{r^2}$

Separate variables: $\int mdvv = \int \frac{-GMm}{r^2}dr$

$$\frac{1}{2}mv^2 = \frac{GMm}{r} + E \text{ where } E \text{ is a constant.}$$

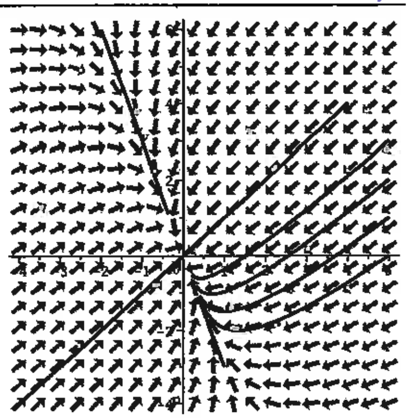
Thus we have derived the physics formula, conservation of energy:

$$\frac{1}{2}mv^2 + \frac{-GMm}{r} = E$$

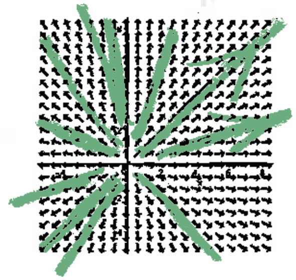
I.e., Kinetic Energy + Potential Energy = constant

$m = -\frac{1}{3}$

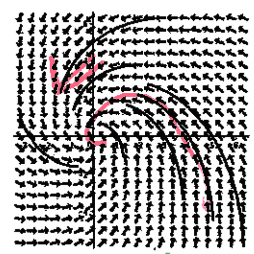
$x' = -4x - y$
$y' = -3x + 2y$



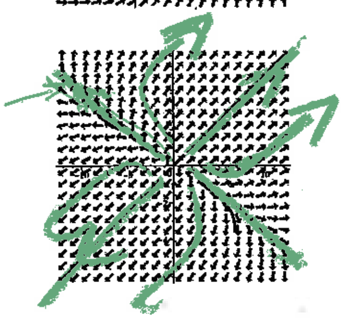
Suppose the following represent direction fields of linear systems of 1rst order differential equations in the phase plane. What can you say about solutions to these systems of equations.



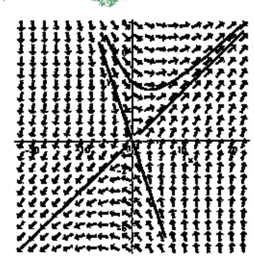
unstable
1 repeated positive real e. value



asym stable
complex e. value $a + bi$
$a < 0$



2 real positive e. value
r_1 w/ e. vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
r_2 w/ e. vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$
unstable



unstable