$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A - \lambda I = \begin{vmatrix} a-\lambda & b \\ -b & a-\lambda \end{vmatrix} = (a-\lambda)^2 + b^2$$
$$= \lambda^2 - 2a\lambda + a^2 + b^2$$

Thus $\lambda = \frac{2a \pm \sqrt{4a^2 - 4(a^2+b^2)}}{2} = \frac{2a \pm \sqrt{-4b^2}}{2} = a \pm bi$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ implies $\begin{matrix} x_1' = ax_1 + bx_2 \\ x_2' = -bx_1 + ax_2 \end{matrix}$

Change to polar coordinates: $r^2 = x_1^2 + x_2^2$ and $tan\theta = \frac{x_2}{x_1}$

Take derivative with respect to $t$ of both equations:

$2rr' = 2x_1x_1' + 2x_2x_2'$ implies

$rr' = x_1(ax_1 + bx_2) + x_2(-bx_1 + ax_2)$

$= ax_1^2 + bx_1x_2 - bx_1x_2 + ax_2^2 = a(x_1^2 + x_2^2) = ar^2$

Thus $rr' = ar^2$ implies $\frac{dr}{dt} = ar$ and thus $\boxed{r = Ce^{at}}$.

$(sec^2\theta)\theta' = \frac{x_1x_2' - x_1'x_2}{x_1^2} = \frac{x_1(-bx_1+ax_2)-(ax_1+bx_2)x_2}{x_1^2}$

$= \frac{-bx_1^2 + ax_1x_2 - ax_1x_2 - bx_2^2}{x_1^2} = \frac{-b(x_1^2+x_2^2)}{x_1^2} = \frac{-b(r^2)}{x_1^2} = -bsec^2\theta$

$(\cancel{sec^2\theta})\theta' = -b\cancel{sec^2}\theta$ implies $\theta' = -b$ and thus $\boxed{\theta = -bt + \theta_0}$

Let ~~$y = Bx$~~ for
$\vec{x} = P\vec{y}$ for change of basis

$\vec{x}' = A\vec{x}$

$[P\vec{y}]' = AP\vec{y}$

$P\vec{y}' = AP\vec{y} \Rightarrow \vec{y}' = (P^{-1}AP)\vec{y}$

if $P$ constant matrix