Engineering Review
Monday 6:30 pm

IVP

Solving first order differential equation:

Method 1 (sect. 2.2): Separate variables.

Method 2 (sect. 2.1): If linear $[y'(t)+p(t)y(t) = g(t)]$, multiply equation by an integrating factor $u(t) = e^{\int p(t)dt}$.

Basics

$$y' + py = g$$
$$y'u + upy = ug$$
$$(uy)' = ug$$
$$\int (uy)' = \int ug$$
$$uy = \int ug$$
etc...

Method 3 (sect. 2.4): Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

y^n y^n y^n

when $n > 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

If $v = \frac{dx}{dt}$, can use the following to simplify (especially if there are 3 variables).

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

integration techniques: u-substitution, integration by parts, partial fractions.

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direction field = slope field = graph of $\frac{dv}{dt}$ in t, v-plane.
*** can use slope field to determine behavior of v including as $t \to \infty$.
Equilibrium Solution = constant solution
stable, unstable, semi-stable.

Ch 1
2.5

Solving second order differential equation:

p. 135: $y'' = f(t, y')$, $y'' = f(y, y')$,

Transform to first order: Let $v = y'$.

If needed, note $v' = \frac{dv}{dt} = \frac{dv}{dt}\frac{dy}{dy} = \frac{dv}{dy}\frac{dy}{dt} = \frac{dv}{dy}v$.

Note this trick sometimes helpful for first order equations.

Ch 3: linear $ay'' + by' + cy = 0$,   3.1

Need to have two independent solutions.

If ϕ_1, ϕ_2 are solutions to a LINEAR HOMOGENEOUS differential equation, $c_1\phi_1 + c_2\phi_2$ is also a solution

$y = c_1\phi_1 + c_2\phi_2$

3.1: $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$