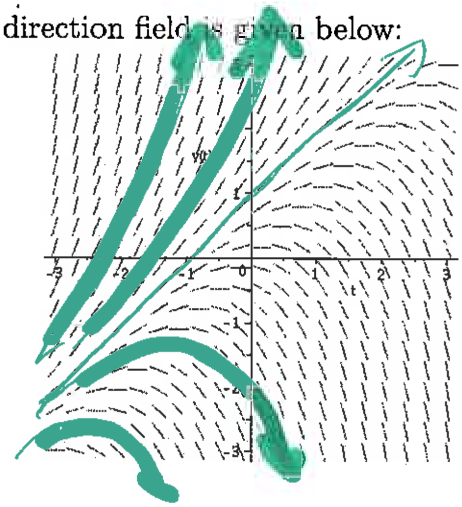
4.) Circle the general solution to the differential equation whose direction field is given below:



A) ~~$y = t + C$~~

B) ~~$y = t^2 + C$~~

C) ~~$y = e^t + C$~~

D) $y = Ce^t + t + 1$

E) $y = Ce^t$

F) ~~$y = e^t + t + C$~~

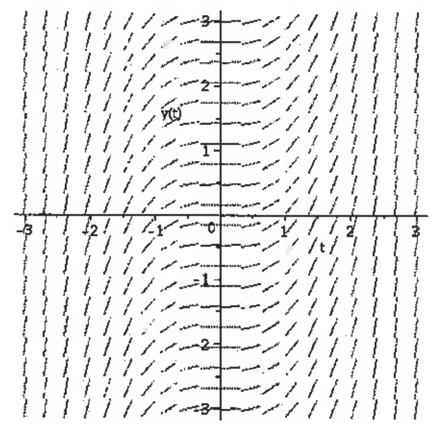
G) ~~$y = ln(t) + C$~~

H) ~~$y = C$~~

I) ~~$y = sin(t) + C$~~

J) ~~$y = cos(t) + C$~~

5.) Which of the following could be the general solution to the differential equation whose direction field is given below:



A) $y = t + C$

B) $y = t^2 + C$

C) $y = e^t + C$

D) $y = \frac{(t-1)^3}{3} + C$

E) $y = Ce^t$

F) $y = \frac{t^3}{3} + C$

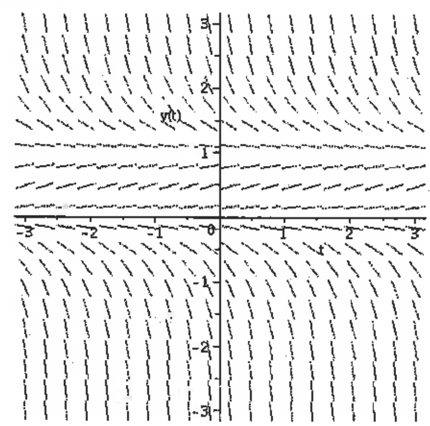
G) $y = ln(t) + C$

H) $y = C$

I) $y = \frac{Ct^3}{3}$

J) $y = \frac{C(t-1)^3}{3}$

6.) Circle the differential equation whose direction field is given below:



A) $y' = t^2$

B) $y' = y + 3$

C) $y' = e^t$

D) $y' = t + 1$

E) $y' = t - y$

F) $y' = y - t$

G) $y' = (1 + y)(1 - y)$

H) $y' = y(1 + y)$

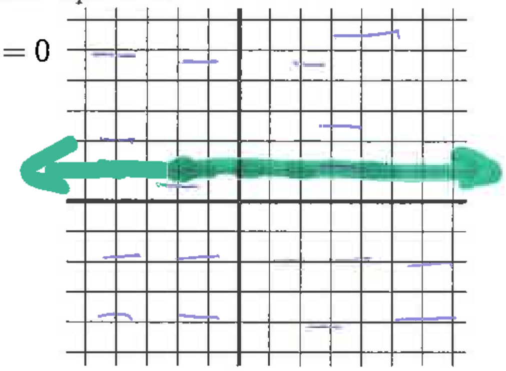
I) $y' = t(1 - t)$

J) $y' = y(1 - y)$

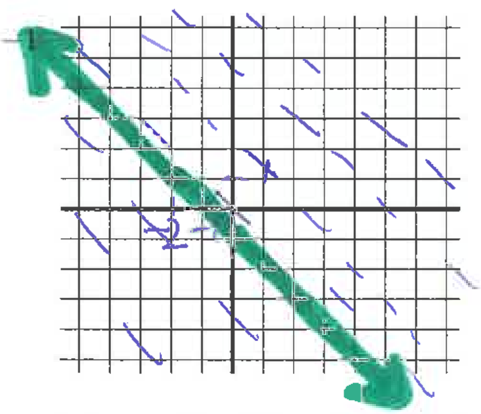
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

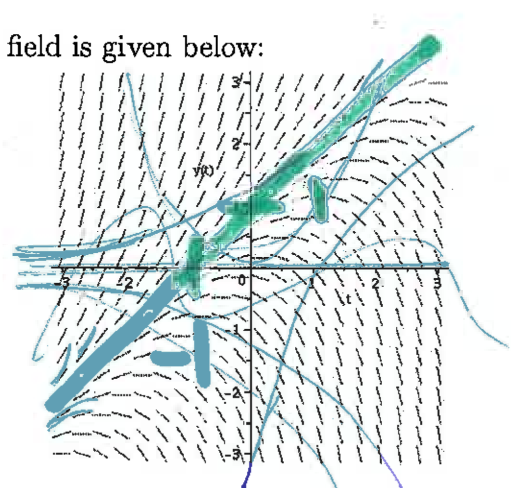
a.) $y' = 0$



b.) $y' = -1$



2.) Circle a solution to the differential equation whose direction field is given below:



A) $y = t^2$

B) $y = \frac{1}{2}t + 1$

C) $y = e^t$

D) $y = t + 1$

E) $y = -2e^t$

F) $y = 2t + 1$

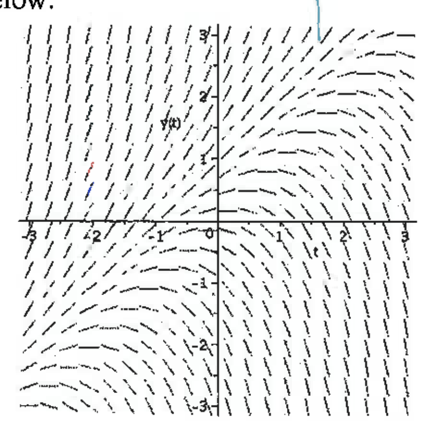
G) $y = ln(t)$

H) $y = 0$

I) $y = sin(t)$

J) $y = cos(t)$

3.) Circle the differential equation whose direction field is given below:



A) $y' = t^2$

B) $y' = \frac{1}{2}t + 1$

C) $y' = e^t$

D) $y' = t + 1$

E) $y' = -2e^t$

F) $y' = y - t$

G) $y' = ln(t)$

H) $y' = 0$

I) $y' = sin(t)$

J) $y' = cos(t)$

Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

I.e., $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$.

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

Suppose f is ~~cont. on (a, b)~~ and the point $t_0 \in (a, b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$ $\Leftarrow$ Calc 1

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt \Rightarrow y = F(t) + C$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$ $\Leftarrow$ unique soln for C

Hence unique solution (if domain connected) to IVP:
$$y = F(t) + y_0 - F(t_0)$$

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

1.1: Direction Fields ********************

*******~~Existence/Uniqueness~~ of solution**********

Thm 2.4.2: Suppose the functions
$z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$
and the point $(t_0, y_0) \in (a, b) \times (c, d)$,
then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$\boxed{y' = f(t, y), \quad y(t_0) = y_0.}$$ IVP

Thm 2.4.1: If p and g are continuous on (a, b) and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on (a, b) that satisfies the following initial value problem:

$$\boxed{y' + p(t)y = g(t), \quad y(t_0) = y_0.}$$ $e^{\int p(t)}$

But in general, $y' = f(t, y)$, solution may or may not exist and solution may or may not be unique.

PRO (http://www.wolframalpha.com/pro) ABOUT (//www.wolfr

slope field

Examples (//www.wolframalpha.com/examples/?src=input) Ranc

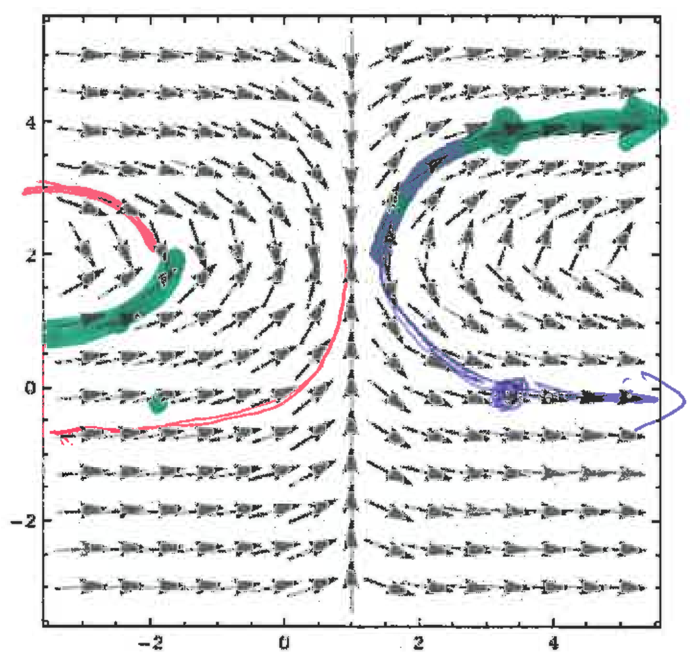
Assuming "slope field" refers to a computation | Use as referring to a mathematical definition instead

vector field:	{1,1/((1-x)(2-y))}/sqrt(
variable 1:	x
lower limit 1:	-3
upper limit 1:	5
variable 2:	y
lower limit 2:	-3
upper limit 2:	5

Input:

$$\text{VectorPlot}\left[\frac{\left\{1, \frac{1}{(1-x)(2-y)}\right\}}{\sqrt{1+\frac{1}{((1-x)(2-y))^2}}}, \{x, -3, 5\}, \{y, -3, 5\}\right]$$

Result:



2.3: Modeling with differential equations.

Ex.: $F = ma = mv'$

$a =$ acceleration $= v' = x''$
$v =$ velocity $= x'$
$x =$ position
$m =$ mass $\qquad mg =$ weight

Model 1: Falling ball near earth, neglect air resistance.

$F_g =$ Gravitational force $= -mg$
IF the positive direction points up.
Note in some examples in the book, the positive direction points down ($F_g = +mg$) while in other examples in the book, the positive direction points up ($F_g = -mg$)

$mv' = -mg$ implies $v' = -g$. Thus $v = -gt + C$.

IVP: $v(0) = v_0$ implies $v_0 = -g(0) + C$ implies $C = v_0$. Thus $v = -gt + v_0$

$x' = v = -gt + v_0$ implies $x = -\frac{1}{2}gt^2 + v_0t + C$.

IVP: $x(0) = x_0$ implies $x_0 = -\frac{1}{2}g(0)^2 + v_0(0) + C$ implies $C = x_0$.

Thus $x = -\frac{1}{2}gt^2 + v_0t + x_0$.

Note when ball reaches maximum height $v = 0$

Model 2: Falling ball near earth, include air resistance.

Let $A(v) =$ the force due to air resistance.

$mv' = F_g + R(v) = -mg + A(v)$

Model 3: Far from earth.

$F_g = -mg\frac{R^2}{(R+x)^2}$ where $R =$ radius of the earth.

If x is small, $\frac{R^2}{(R+x)^2} \sim 1$ and thus $F_g = -mg$ when close to earth.

For large x, $mv' = -mg\frac{R^2}{(R+x)^2}$ where R constant.

$\frac{dv}{dt} = -mg\frac{R^2}{(R+x)^2}$ with 3 variables: v, t, x

To eliminate one variable: $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

Note this trick can also be used to simplify some problems.

$\Rightarrow \frac{dv}{dt} = v\frac{dv}{dx}$