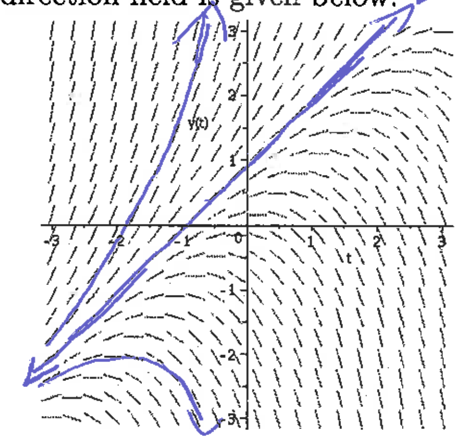
$C>0$ $C=0$

4.) Circle the general solution to the differential equation whose direction field is given below:



$\cancel{\textbf{A)}\ y = t + C}$ $\cancel{\textbf{B)}\ y = t^2 + C}$

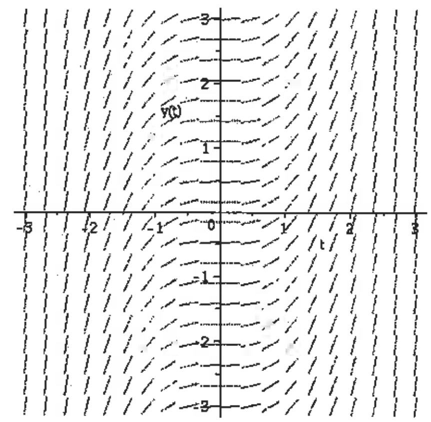
**C)** $\cancel{y = e^t + C}$ **D)** $y = Ce^t + t + 1$

$\cancel{\textbf{E)}\ y = Ce^t}$ $\cancel{\textbf{F)}\ y = e^t + t + C}$

$\cancel{\textbf{G)}\ y = ln(t) + C}$ $\cancel{\textbf{H)}\ y = C}$

$\cancel{\textbf{I)}\ y = sin(t) + C}$ $\cancel{\textbf{J)}\ y = cos(t) + C}$

5.) Which of the following could be the general solution to the differential equation whose direction field is given below:



**A)** $y = t + C$ **B)** $y = t^2 + C$

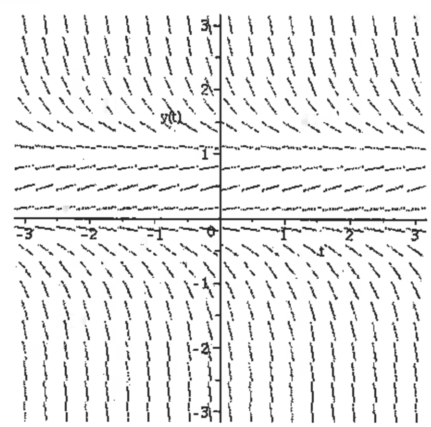
**C)** $y = e^t + C$ **D)** $y = \frac{(t-1)^3}{3} + C$

**E)** $y = Ce^t$ **F)** $y = \frac{t^3}{3} + C$

**G)** $y = ln(t) + C$ **H)** $y = C$

**I)** $y = \frac{Ct^3}{3}$ **J)** $y = \frac{C(t-1)^3}{3}$

6.) Circle the differential equation whose direction field is given below:



**A)** $y' = t^2$ **B)** $y' = y + 3$

**C)** $y' = e^t$ **D)** $y' = t + 1$

**E)** $y' = t - y$ **F)** $y' = y - t$

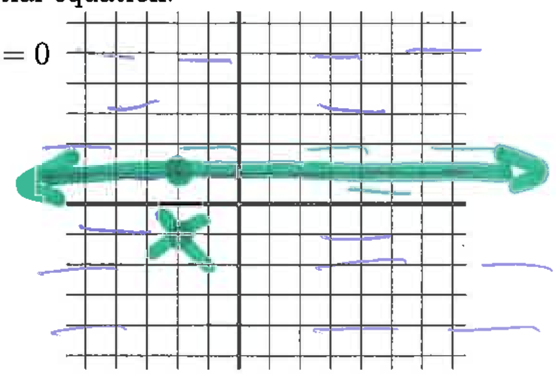
**G)** $y' = (1 + y)(1 - y)$ **H)** $y' = y(1 + y)$

**I)** $y' = t(1 - t)$ **J)** $y' = y(1 - y)$

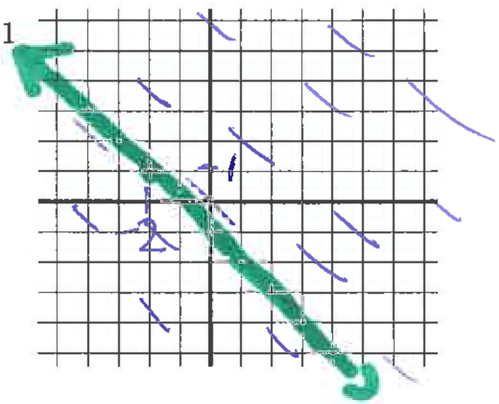
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

a.) $y' = 0$



b.) $y' = -1$



2.) Circle a solution to the differential equation whose direction field is given below:

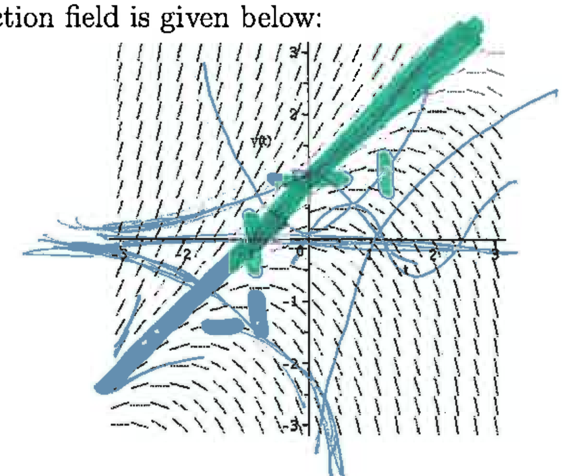
A) $y = t^2$  B) $y = \frac{1}{2}t + 1$

C) $y = e^t$  D) $y = t + 1$

E) $y = -2e^t$  F) $y = 2t + 1$

G) $y = ln(t)$  H) $y = 0$

I) $y = sin(t)$  J) $y = cos(t)$



3.) Circle the differential equation whose direction field is given below:

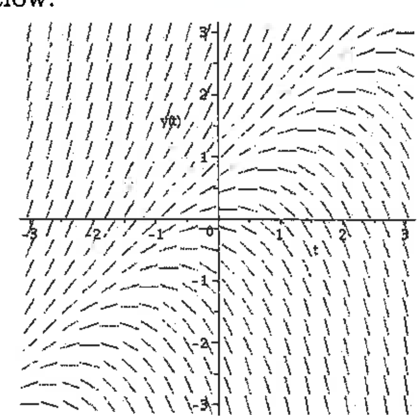
A) $y' = t^2$  B) $y' = \frac{1}{2}t + 1$

C) $y' = e^t$  D) $y' = t + 1$

E) $y' = -2e^t$  F) $y' = y - t$

G) $y' = ln(t)$  H) $y' = 0$

I) $y' = sin(t)$  J) $y' = cos(t)$



Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose $f$ continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

I.e., $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$.

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where $F$ is any antiderivative of $f$, that is $F' = f$.

Suppose $f$ is cont. on $(a, b)$ and the point $t_0 \in (a, b)$,
Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$ ← Calc 1

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt \quad \Rightarrow y = F(t) + C$$

$y = F(t) + C$ where $F$ is any anti-derivative of $F$.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$ ← unique soln for C

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$



CH 2: Solve $\frac{dy}{dt} = f(t, y)$

---

***1.1: Direction Fields ************************

---

*******Existence/Uniqueness of solution**********

Thm 2.4.2: Suppose the functions $z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$, then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$\boxed{y' = f(t, y), \quad y(t_0) = y_0.}$$ IVP

Thm 2.4.1: If $p$ and $g$ are continuous on $(a, b)$ and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on $(a, b)$ that satisfies the following initial value problem:

$$\boxed{y' + p(t)y = g(t), \quad y(t_0) = y_0.}$$

$e^{\int p(t)}$

---

But in general, $y' = f(t, y)$, solution may or may not exist and solution may or may not be unique.

slope field

Examples (//www.wolframalpha.com/examples/?src=input) Ranc

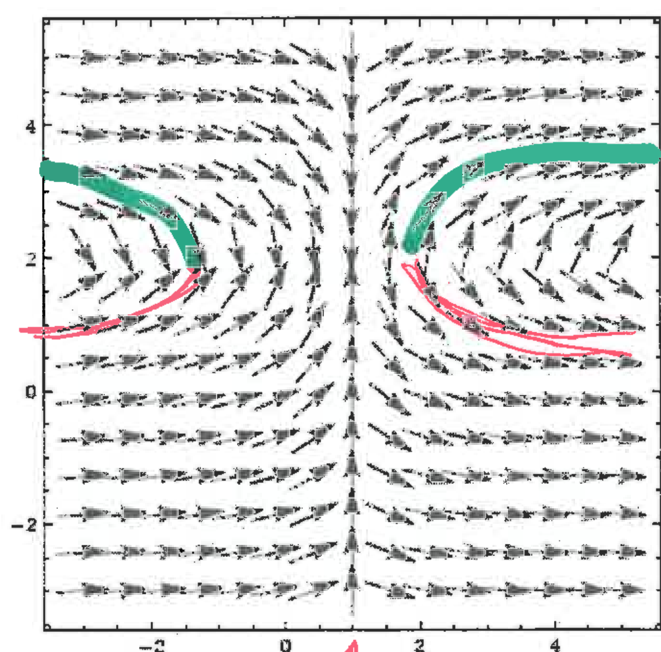
Assuming "slope field" refers to a computation | Use as referring to a mathematical definition instead

- vector field: {1,1/((1-x)(2-y))}/sqrt(
- variable 1: x
- lower limit 1: -3
- upper limit 1: 5
- variable 2: y
- lower limit 2: -3
- upper limit 2: 5

Input:

$$\text{VectorPlot}\left[\frac{\left\{1, \frac{1}{(1-x)(2-y)}\right\}}{\sqrt{1+\frac{1}{((1-x)(2-y))^2}}}, \{x, -3, 5\}, \{y, -3, 5\}\right]$$

Result:




$y = 2$

$t = 1$