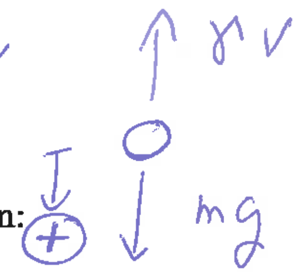
Let x = distance

$v = \frac{dx}{dt}$

1rst order



1.1: Examples of differentiable equation:

1.) $F = ma = m\frac{dv}{dt} = mg - \gamma v$

2.) Mouse population increases at a rate proportional to the current population:

simplest model $\frac{dp}{dt} = rp$

More general model : $\frac{dp}{dt} = rp - k$

where $p(t)$ = mouse population at time t,

r = growth rate or rate constant,

k = predation rate = # mice killed per unit time.

1.1

direction field = slope field = graph of $\frac{dv}{dt}$ in t, v-plane.

www.math.rutgers.edu/~sontag/JODE/JOdeApplet.html

*** can use slope field to determine behavior of v including as $t \to \infty$. ← long term behaviour

Equilibrium Solution = constant solution ← when it exists

2.2 = 1.2: Solve $\frac{dy}{dt} = ay + b$ by separating variables:

$\frac{dy}{ay+b} = dt$ lost $y = \frac{-b}{a}$ sol'n

$\int \frac{dy}{ay+b} = \int dt$ implies $\frac{ln|ay+b|}{a} = t + C$

$ln|ay+b| = at + C$ implies $e^{ln|ay+b|} = e^{at+C}$

$|ay+b| = e^C e^{at}$ implies $ay + b = \pm(e^C e^{at})$

$ay = Ce^{at} - b$ implies $y = Ce^{at} - \frac{b}{a}$

Initial Value Problem: $y(t_0) = y_0$

gained it for $C=0$

1.3:

ODE (ordinary differential equation): single independent variable

Ex: $\frac{dy}{dt} = ay + b$

vs

PDE (partial differential equation): several independent variables

Ex: $\frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$

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order of differential eq'n: order of highest derivative

example of order n: $y^{(n)} = f(t, y, ..., y^{(n-1)})$

$a_0(t) = 0 \Rightarrow n$th order

$\boxed{\text{Linear vs Non-linear}}$

linear: $a_0(t)y^{(n)} + ... + a_n(t)y = g(t)$

Determine if linear or non-linear:

2nd order

Ex: $ty'' - t^3y' - 3y = sin(t)$ ← linear

Ex: $2y'' - 3y' - 3y^2 = 0$ ← non linear

********Existence of a solution**************

********Uniqueness of solution***************

CH 2: Solve $\frac{dy}{dt} = f(t, y)$ ← usually easiest

1.2 = 2.2: Separation of variables: $N(y)dy = P(t)dt$

2.1: First order linear eqn: $\frac{dy}{dt} + p(t)y = g(t)$

1st order linear ODE

Ex 1: $t^2y' + 2ty = tsin(t)$

Ex 2: $y' = ay + b$ → $\int \frac{dy}{ay+b} = \int dt$ 2.2

Ex 3: $y' + 3t^2y = t^2$, $y(0) = 0$

Note: could use section 2.2 method, separation of variables to solve ex 2 and 3.

$\frac{dy}{dt} = (t^2 - 3t^2y)dt$

$dy = (1-3y)t^2dt \Rightarrow \int \frac{dy}{1-3y} = \int t^2 dt$ ← 2.2 & 2.1

$\cancel{t^2}\frac{dy}{dt} = \frac{(\sin t - 2ty)dt}{t^2}$ ← cannot separate variables

Ex 1: $t^2y' + 2ty = sin(t)$

(note, cannot use separation of variables). Must use 2.1 methods

product rule

$t^2y' + 2ty = sin(t)$

$(t^2y)' = sin(t)$

$\int (t^2y)'dt = \int sin(t)dt$

$(t^2y) = -cos(t) + C$ implies $y = -t^{-2}cos(t) + Ct^{-2}$

Gen ex: Solve $y' + p(x)y = g(x)$

Let $F(x)$ be an anti-derivative of $p(x)$

$e^{F(x)}y' + [p(x)e^{F(x)}]y = g(x)e^{F(x)}$

$e^{F(x)}y' + [F'(x)e^{F(x)}]y = g(x)e^{F(x)}$

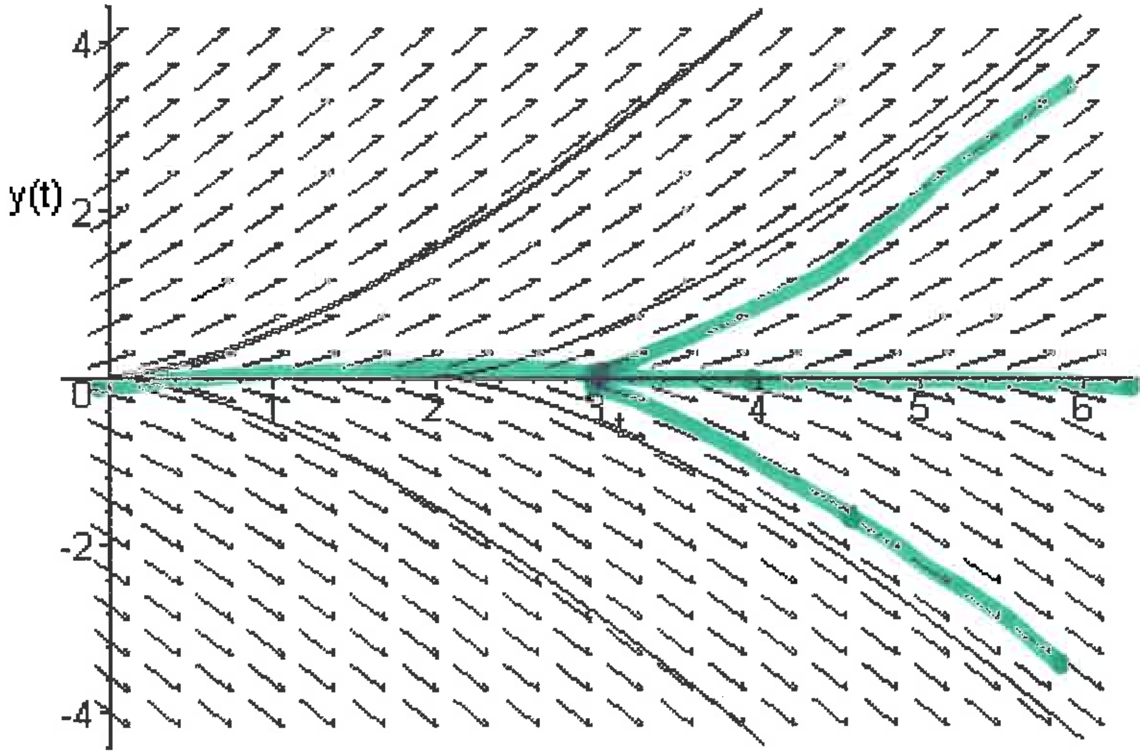
$[e^{F(x)}y]' = g(x)e^{F(x)}$

$e^{F(x)}y = \int g(x)e^{F(x)}dx$

$y = e^{-F(x)} \int g(x)e^{F(x)}dx$

http://homepage.math.uiowa.edu/~idarcy/COURSES/100/SPRING15/3600.html

$y' = y^{1/3}$



not horizontal asymptote

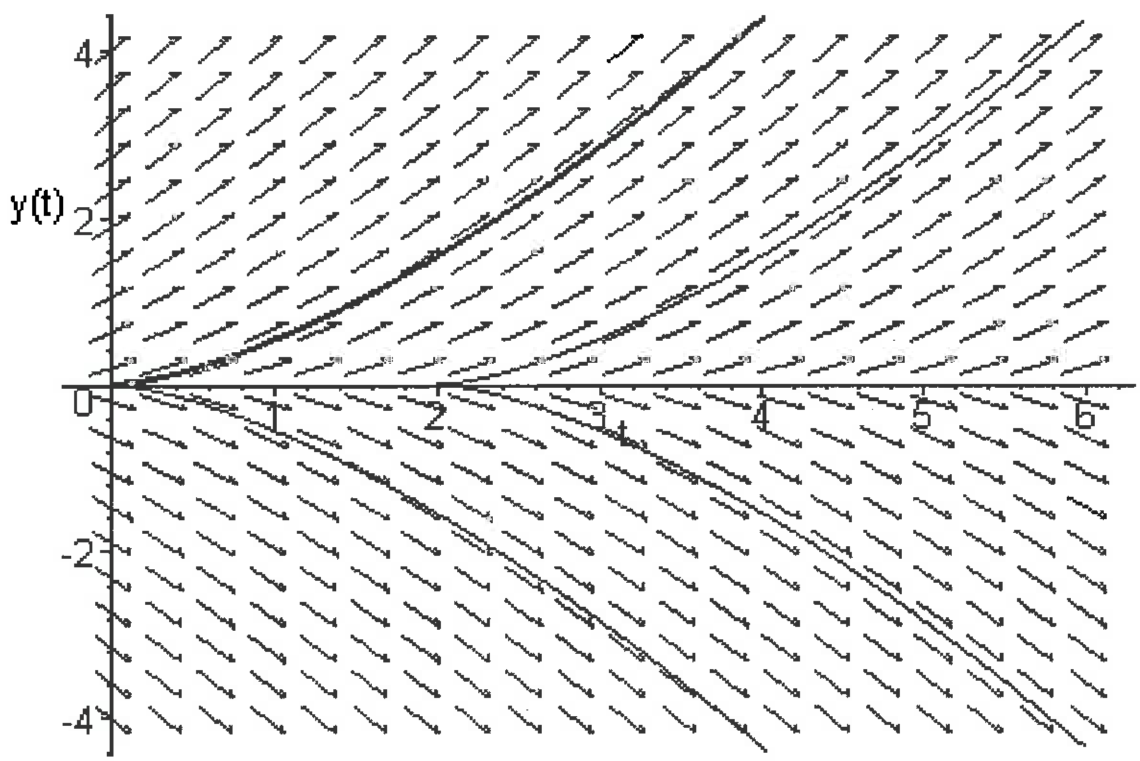


Figure 2.4.1 from ***Elementary Differential Equations and Boundary Value Problems*, Eighth Edition** by William E. Boyce and Richard C. DiPrima