$a_0(t) \neq 0 \Rightarrow$ n^{th} order diff eqn

Linear vs Non-linear

linear: $a_0(t)\boxed{y^{(n)}} + ... + a_n(t)\boxed{y} = g(t)$

Determine if linear or non-linear:

2nd order

Ex: $t\boxed{y''} - t^3\boxed{y'} - 3\boxed{y} = sin(t)$

Ex: $2y'' - 3y' - 3y^2 = 0$ Not linear

********Existence of a solution**************

********Uniqueness of solution***************

CH 2: Solve $\frac{dy}{dt} = f(t,y)$

usually easiest

1.2 = 2.2: Separation of variables: $N(y)dy = P(t)dt$

2.1: First order linear eqn: $\frac{dy}{dt} + p(t)y = g(t)$

2.2 = 1.2

linear

Ex 1: $t^2y' + 2ty = tsin(t)$

Ex 2: $y' = ay + b$

$\frac{dy}{ay+b} = dt$

Ex 3: $y' + 3t^2y = t^2$, $y(0) = 0$

Note: could use section 2.2 method, separation of variables to solve ex 2 and 3.

$\frac{dy}{\cancel{dt}} = (t^2 - 3t^2y)dt$

$dy = (1-3y)t^2dt \rightarrow \int \frac{dy}{1-3y} = \int t^2 dt$

$t^2\frac{dy}{dt} + 2ty = \sin t$

$dy = \frac{(\sin t - 2ty)dt}{t^2}$

Ex 1: $t^2y' + 2ty = sin(t)$
(note, cannot use separation of variables).

Cannot use separation of variables

$t^2y' + 2ty = sin(t)$

$(t^2y)' = sin(t)$

$\int (t^2y)'dt = \int sin(t)dt$

$(t^2y) = -cos(t) + C$ implies $y = -t^{-2}cos(t) + Ct^{-2}$

Gen ex: Solve $y' + p(x)y = g(x)$

Let $F(x)$ be an anti-derivative of $p(x)$

$e^{F(x)}y' + [p(x)e^{F(x)}]y = g(x)e^{F(x)}$

$e^{F(x)}y' + [F'(x)e^{F(x)}]y = g(x)e^{F(x)}$

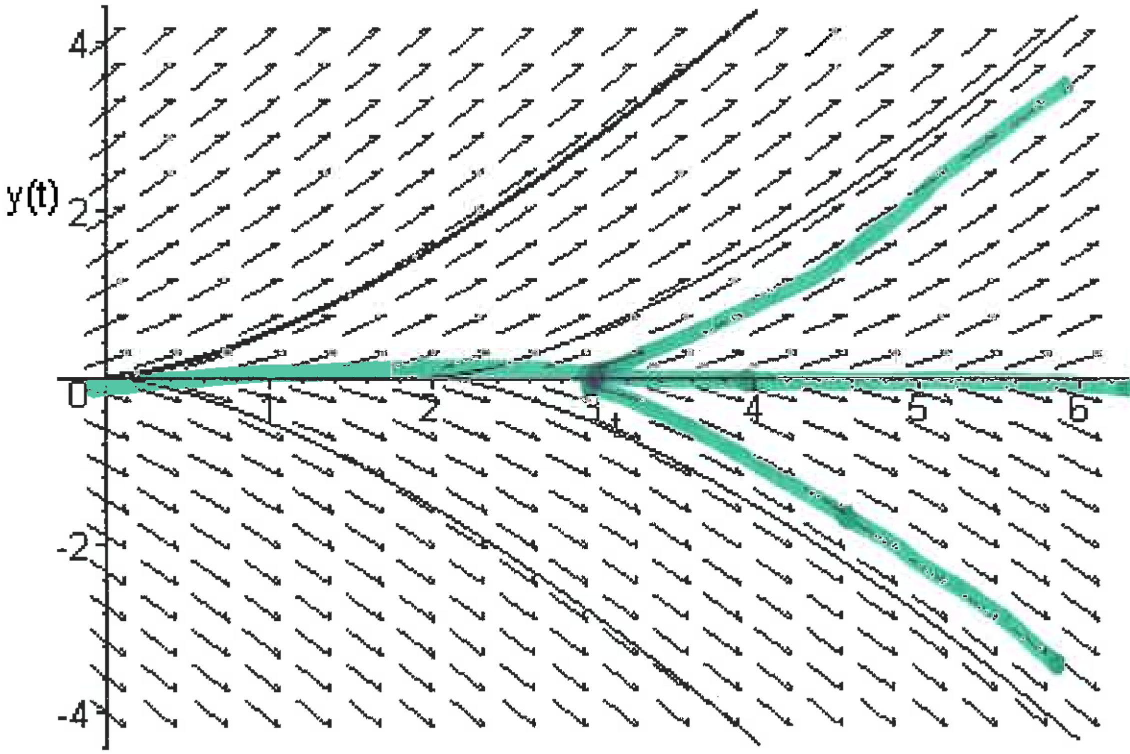
$[e^{F(x)}y]' = g(x)e^{F(x)}$

$e^{F(x)}y = \int g(x)e^{F(x)}dx$

$y = e^{-F(x)} \int g(x)e^{F(x)}dx$

http://homepage.math.uiowa.edu/~idarcy/COURSES/100/SPRING15/3600.html

$$y' = y^{1/3}$$



not horizontal asymptote

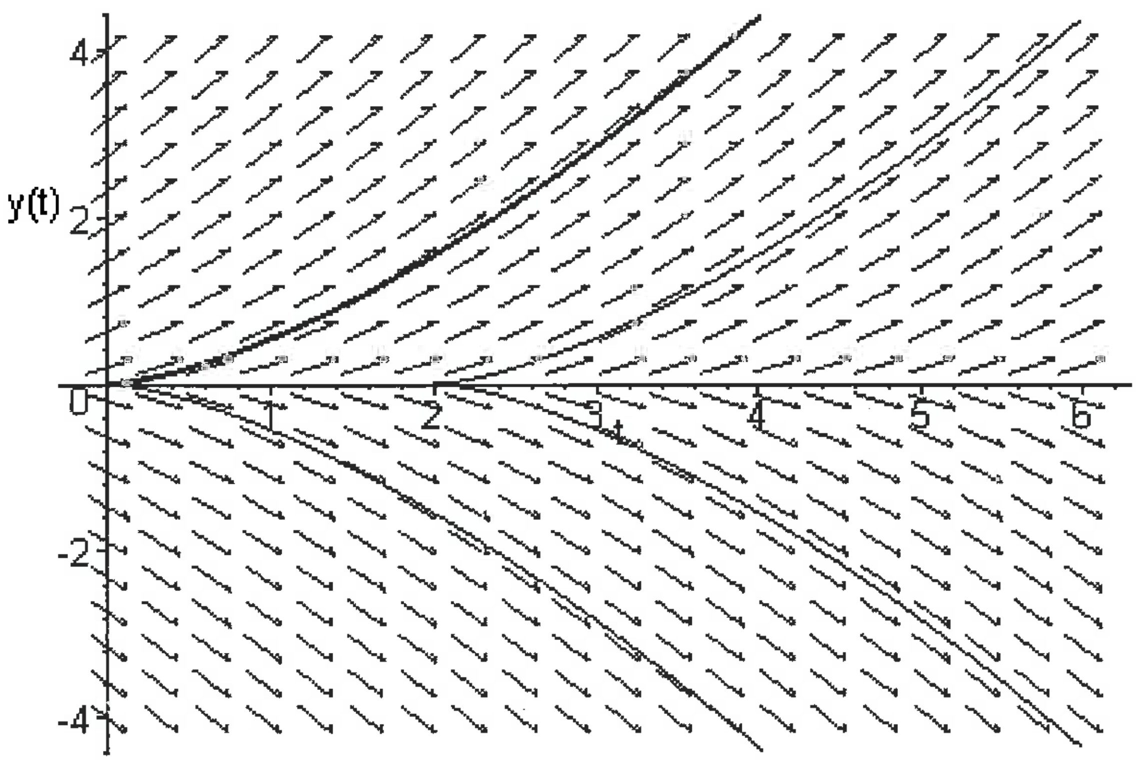


Figure 2.4.1 from ***Elementary Differential Equations and Boundary Value Problems*, Eighth Edition** by William E. Boyce and Richard C. DiPrima