22M:100:001 (MATH:3600:0001) Quiz 2
April 3, 2013

1.) Determine which of the following sets of vectors are linearly dependent versus linearly independent. Circle the correct answer

[10] 1i.) $\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \end{pmatrix}\}$

a.) linearly dependent

The span of these three vectors is 2-dimensional. Any collection of more than two vectors spanning a 2-dimensional space must be linearly dependent

[10] 1ii.) $\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}\}$

a.) linearly dependent

The second vector is a multiple of the first vector.

[10] 1iii.) $\{\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}\}$

b.) linearly independent

There are only two vectors and the second vector is NOT a multiple of the first vector.

[10] 1iv.) $\{\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}\}$

a.) linearly dependent

$$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 4 & 2 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -2 & 2 \end{pmatrix}$$

[10] 1v.) $\{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}\}$

b.) linearly independent

Note the first two vectors span a 2-dimensional space that does not contain the third vector. Hence these 3 vectors are linearly independent.

Alternatively $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 0 & 0 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 6 \end{pmatrix}$

2.) If $A\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}5\\5\end{pmatrix}$, $A\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}4\\4\end{pmatrix}$, $A\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}6\\6\end{pmatrix}$, $A\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}8\\8\end{pmatrix}$, $A\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}7\\7\end{pmatrix}$.

[10] 2a.) An eigenvalue of A is _3_

[15] 2b.) 4 eigenvectors corresponding to this eigenvalue are $\begin{pmatrix}2\\2\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\end{pmatrix}, \begin{pmatrix}\pi\\\pi\end{pmatrix}$

Any non-zero scalar multiple of $\begin{pmatrix}2\\2\end{pmatrix}$ is an eigenvector of A with eigenvalue 3.

FYI: $A\begin{pmatrix}1\\0\end{pmatrix}=A\left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right]=A\begin{pmatrix}2\\2\end{pmatrix}-A\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}6\\6\end{pmatrix}-\begin{pmatrix}5\\5\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$

$A\begin{pmatrix}0\\1\end{pmatrix}=A\left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}2\\1\end{pmatrix}\right]=A\begin{pmatrix}2\\2\end{pmatrix}-A\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}6\\6\end{pmatrix}-\begin{pmatrix}4\\4\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$

Thus $A=\begin{pmatrix}1&2\\1&2\end{pmatrix}$. Since the columns (and similarly the rows) are not linearly independent, 0 is also an eigenvalue of A.

Note $A=\begin{pmatrix}1&2\\1&2\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}=0\begin{pmatrix}2\\-1\end{pmatrix}$

Thus an alternate answer is

[10] 2a.) An eigenvalue of A is _0_

[15] 2b.) 4 eigenvectors corresponding to this eigenvalue are $\begin{pmatrix}2\\-1\end{pmatrix}, \begin{pmatrix}-2\\1\end{pmatrix}, \begin{pmatrix}-4\\2\end{pmatrix}, \begin{pmatrix}2\pi\\-\pi\end{pmatrix}$

Any non-zero scalar multiple of $\begin{pmatrix}2\\-1\end{pmatrix}$ is an eigenvector of A with eigenvalue 0.

[15] 3a.) Find the eigenvalues of $A=\begin{pmatrix}1&1\\2&-1\end{pmatrix}$

[10] 3b.) Find one eigenvector corresponding to each eigenvalue.

$|A-rI|=\begin{vmatrix}1-r&1\\2&-1-r\end{vmatrix}=(1-r)(-1-r)-2=r^2-3=0$. Thus $r=\pm\sqrt{3}$

$$A-rI=\begin{pmatrix}1-(\pm\sqrt{3})&1\\2&-1-(\pm\sqrt{3})\end{pmatrix}=\begin{pmatrix}1\mp\sqrt{3}&1\\2&-1\mp\sqrt{3}\end{pmatrix}$$

Note $\begin{pmatrix}1\mp\sqrt{3}&1\\2&-1\mp\sqrt{3}\end{pmatrix}\begin{pmatrix}1\\-1\pm\sqrt{3}\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

Thus a nonzero solution to $(A-rI)\mathbf{x}=\mathbf{0}$ is $\begin{pmatrix}1\\-1\pm\sqrt{3}\end{pmatrix}$

An e. value of A is _$\sqrt{3}$_ & an e. vector corresponding to this e. value is $\begin{pmatrix}1\\-1+\sqrt{3}\end{pmatrix}$

An e. value of A is _$-\sqrt{3}$_ & an e. vector corresponding to this e. value is $\begin{pmatrix}1\\-1-\sqrt{3}\end{pmatrix}$