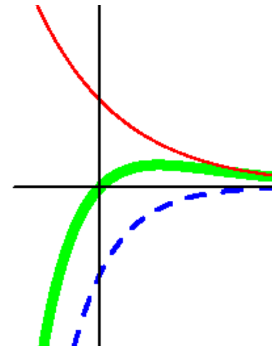
1. The following shows the graphs of $u = e^{-t}$, $u = -e^{-2t}$ and $u = e^{-t} - e^{-2t}$.



1a.) The thin red curve corresponds to ________________.

1b.) The medium dotted blue curve corresponds to ________________.

1c.) The thick light green curve corresponds to ________________.

1d.) $\lim_{t\to-\infty} e^{-t} - e^{-2t} =$

1e.) $\lim_{t\to\infty} e^{-t} - e^{-2t} =$

1f.) As $t \to -\infty$ which term of $e^{-t} - e^{-2t}$ dominates. In other words, does $e^{-t} - e^{-2t}$ look more like e^{-t} or $-e^{-2t}$ for large negative values of t

1f.) As $t \to \infty$ which term of $e^{-t} - e^{-2t}$ dominates. In other words, does $e^{-t} - e^{-2t}$ look more like e^{-t} or $-e^{-2t}$ for large positive values of t

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2a.) Suppose $u(t) = c_1\phi_i + c_1\phi_2$ is the general **homogeneous** solution to a differential equation modeling mechanical vibration **with damping**, then $\lim_{t\to\infty} u(t) =$

2b.) Will the initial values affect the long-term behaviour of the general **homogeneous** solution to a differential equation modeling mechanical vibration **with damping**?

3.) Suppose $u'' + 4u = tcos(2t)$ models mechanical vibration (with no damping and with external force $= tcos(2t)$)

Normally when solving $au'' + bu' + cu = tcos(2t)$ one would guess a non-homogenous solution to be of the form $u = (At + B)[Ccos(2t) + Dsin(2t)]$ since $tcos(2t)$ is the product of a degree 1 polynomial and $cos(2t)$. But that won't work in this case.

$(At + B)[Ccos(2t) + Dsin(2t)] = (At)[Ccos(2t) + Dsin(2t)] + (B)[Ccos(2t) + Dsin(2t)]$

3a.) $u = (B)[Ccos(2t)+Dsin(2t)]$ is NOT a solution to the non-homogeneous equation $u''+4u = tcos(2t)$ since it is a solution to the equation:

Sidenote 1: If you did include this in your guess, this term would cancel out since it is a homogeneous solution, and you would see that any choice of B would work IF the rest of your guess worked. $B = 0$ is the simplest choice.

3b.) $u = (At)[Ccos(2t)+Dsin(2t)]$ is NOT a solution to the non-homogeneous equation $u''+4u = tcos(2t)$ since it is a solution to the equation:

Sidenote 2: If I were solving the equation that answers 3B, I would let C and D swallow the constant A and thus guess $u = t[Ccos(2t) + Dsin(2t)]$ instead of $u = (At)[Ccos(2t) + Dsin(2t)]$

3c.) Thus the best guess for a non-homogeneous solution to $u'' + 4u = tcos(2t)$ would be

Hint: Your answer will likely include 4 unknowns.

3d.) Plugging in $u = t(At + B)[Ccos(2t) + Dsin(2t)]$ would result in 4 nonlinear equations involving products such as AC. Note

$$\begin{aligned} u &= t(At + B)[Ccos(2t) + Dsin(2t)] \\ &= ACt^2cos(2t) + ADt^2sin(2t) + BCtcos(2t) + BDtsin(2t) \\ &= \quad A_1t^2cos(2t) + A_2t^2sin(2t) + A_3tcos(2t) + A_4tsin(2t) \end{aligned}$$

Thus we can instead plug in $u = A_1t^2cos(2t) + A_2t^2sin(2t) + A_3tcos(2t) + A_4tsin(2t)$.

Solve $u'' + 4u = tcos(2t)$ using that

$$[A_1t^2cos(2t) + A_2t^2sin(2t) + A_3tcos(2t) + A_4tsin(2t)]'' \\ +4[A_1t^2cos(2t) + A_2t^2sin(2t) + A_3tcos(2t) + A_4tsin(2t)]$$

$$= -8A_1tsin(2t) + 2A_2sin(2t) - 4A_3sin(2t) + 8A_2tcos(2t) + 2A_1cos(2t) + 4A_4cos(2t)$$

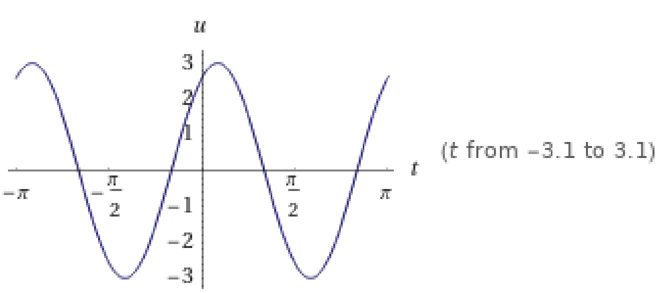
Note: I have done the tedious plugging in and partially simplifying. All you need to do now is determine A_i for $i = 1, 2, 3, 4$ (so this is a short problem – but with a longish answer).

Sidenote 3: If you think about how sin and cos behave when taking derivatives, you might be able to figure out why 2 of the terms are not needed; but most people would guess the above for the solution as determining the best guess can take more work than plugging in. All one needs is one non-homogeneous solution, and the above guess gives us that.

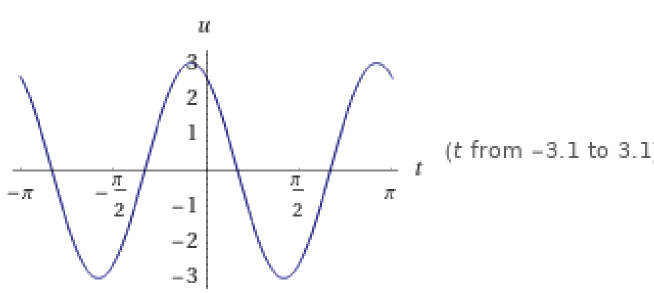
4.) The following are solutions to a second order differential equation modeling mechanical vibration. Match these equations to their graphs. State whether the graph corresponds to an undamped, underdamped, critically damped, or overdamped mechanical vibration.

Equation	Graph	damping
$u = 3cos(2t + \frac{\pi}{6})$		
$u = 3cos(2t - \frac{\pi}{6})$		
$u = 3e^{-2t}cos(2t + \frac{\pi}{6})$		
$u = 3e^{-2t}cos(2t - \frac{\pi}{6})$		
$u = 3e^{-2t}cos(5t + \frac{\pi}{6})$		
$u = 3e^{-2t}cos(5t - \frac{\pi}{6})$		
$u = e^{-2t}(1 + t)$	G	
$u = 100e^{-2t} + e^{-5t}$		
$u = -100e^{-2t} - e^{-5t}$		
$u = -100e^{-2t} + e^{-5t}$		
$u = 100e^{-2t} - e^{-5t}$		

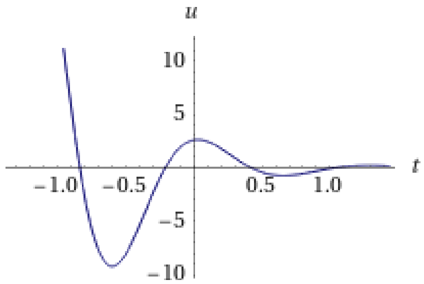
A.



B.

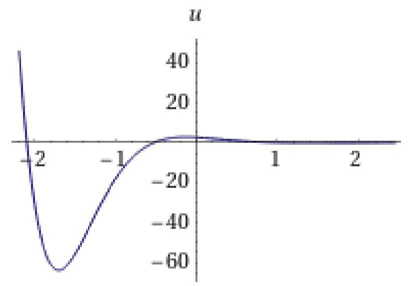


C.



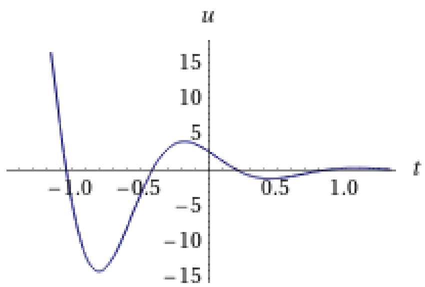
(t from –1.3 to 1.4)

D.



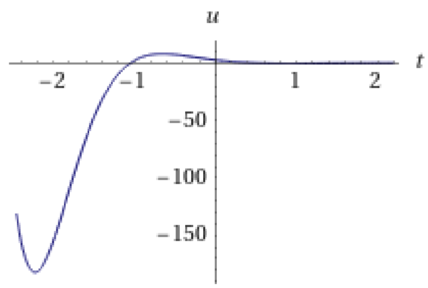
(t from –2.2 to 2.5)

E.



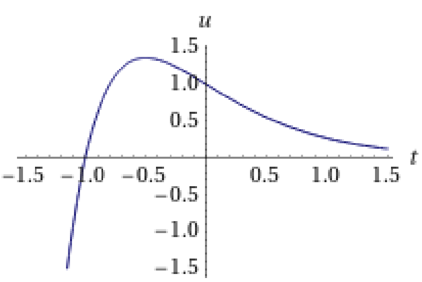
(t from –1.4 to 1.3)

F.



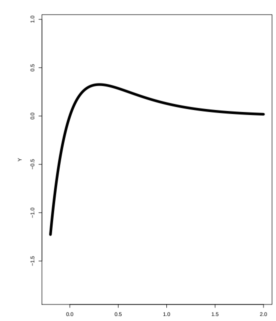
(t from –2.5 to 2.2)

G.

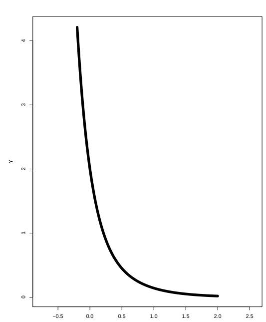


(t from –1.5 to 1.5)

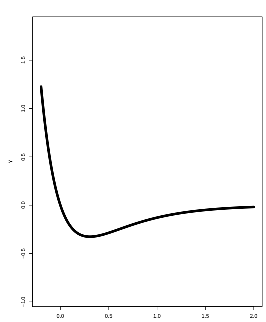
H.



I.



J.



K.