Slope Field Review

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$0 = (3-3)(3+1)$ ✓

Standard slope field example: $\boxed{y' = (y-3)(y+1)}$

Equilibrium solution = constant solution.

$$y = C \quad \text{iff} \quad y' = 0$$

Thus to find equilibrium solution(s) if there are any, set $y' = 0$:

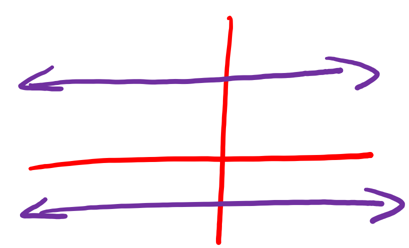
$y' = 0$

$0 = (y-3)(y+1)$ implies $y = 3$ and $y = -1$

Since these are constant functions, the equilibrium solutions are $y = 3$ and $y = -1$.

$\cancel{y = 3, -1}$

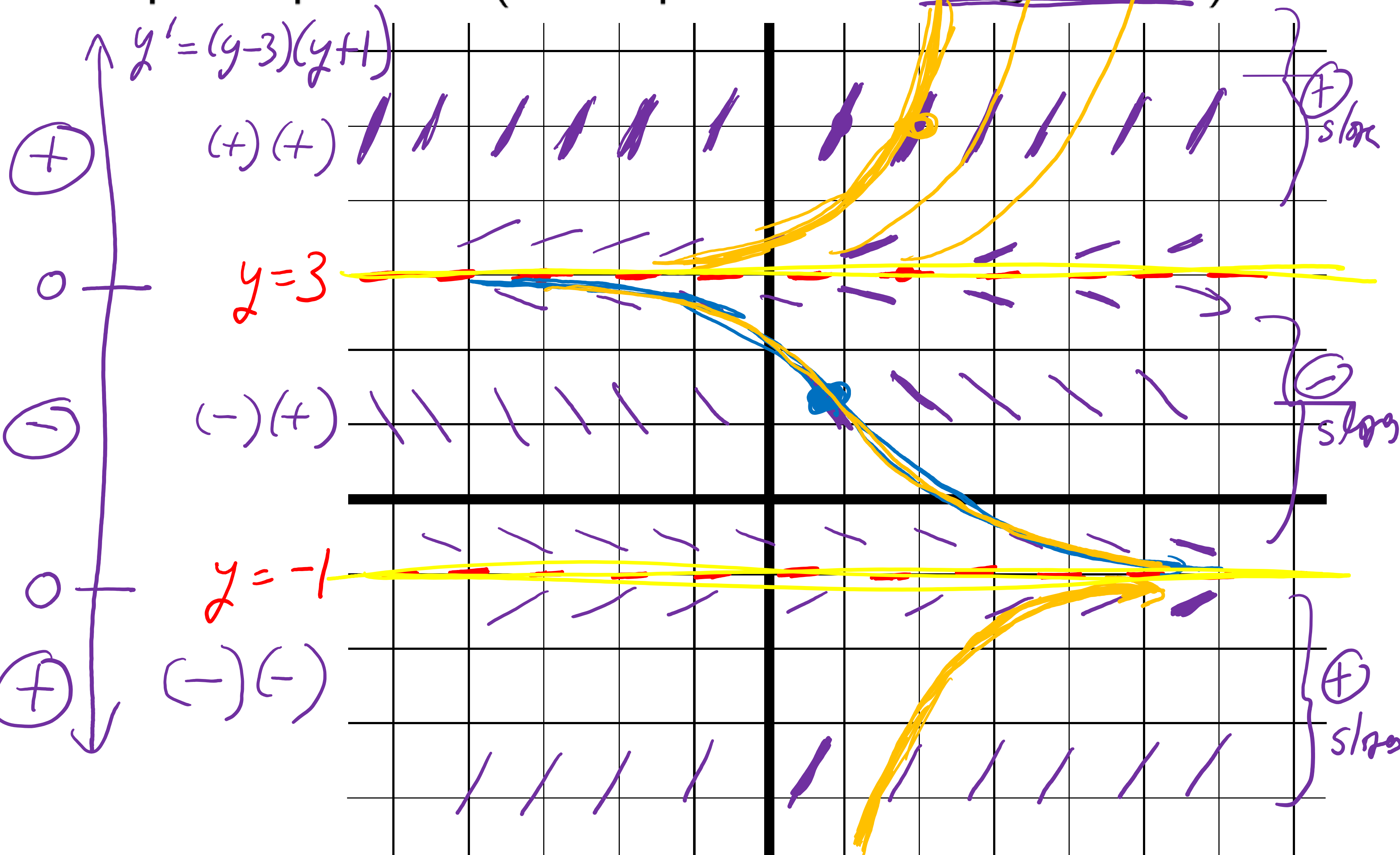
$y = 3$ and $y = -1$



Standard slope field example: $\boxed{y' = (y-3)(y+1)}$

direction field

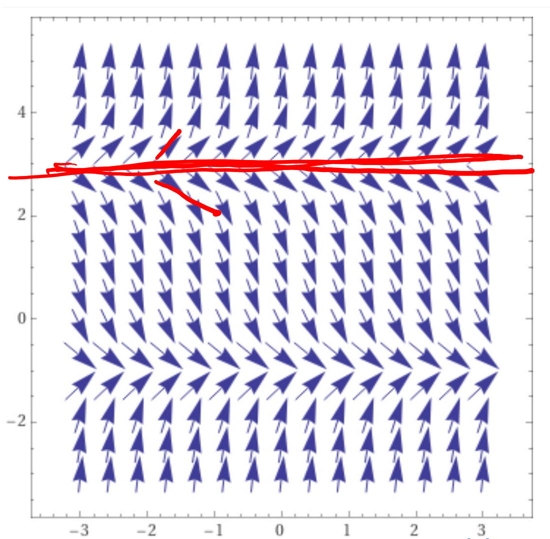
Graph slope field (small portion of tangent line):



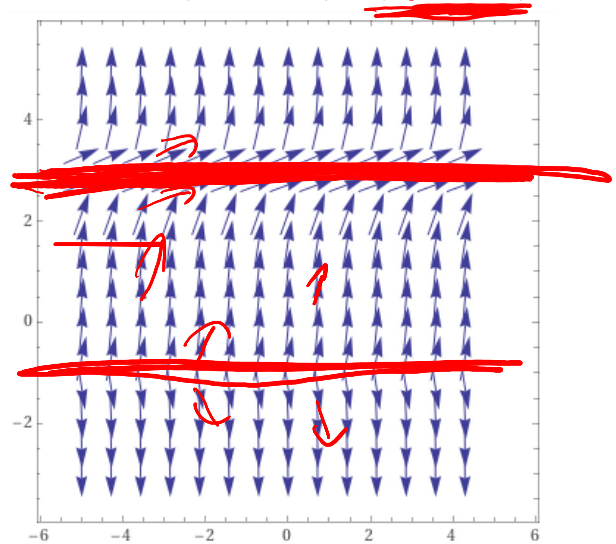
If $y' = f(x)$ is a piecewise continuous function, the slope can only change from postive to negative and vice versa by passing thru

- a slope of 0 (horizontal tangent line) or
- a slope of ∞ (vertical tangent line) or jump discontinuity undefined.

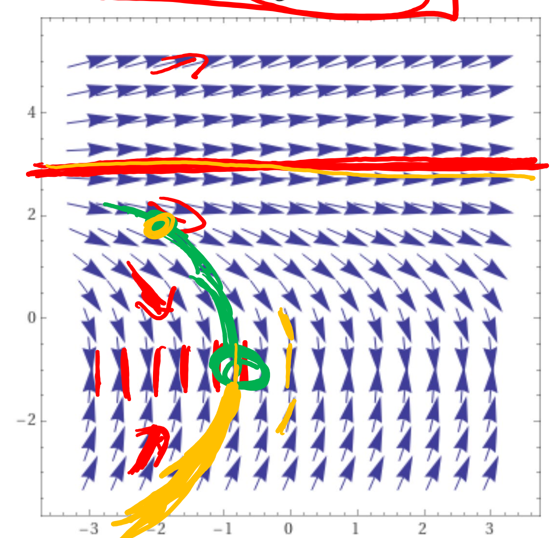
$y' = (y-3)(y+1)$



$y' = (y-3)^2(y+1)$



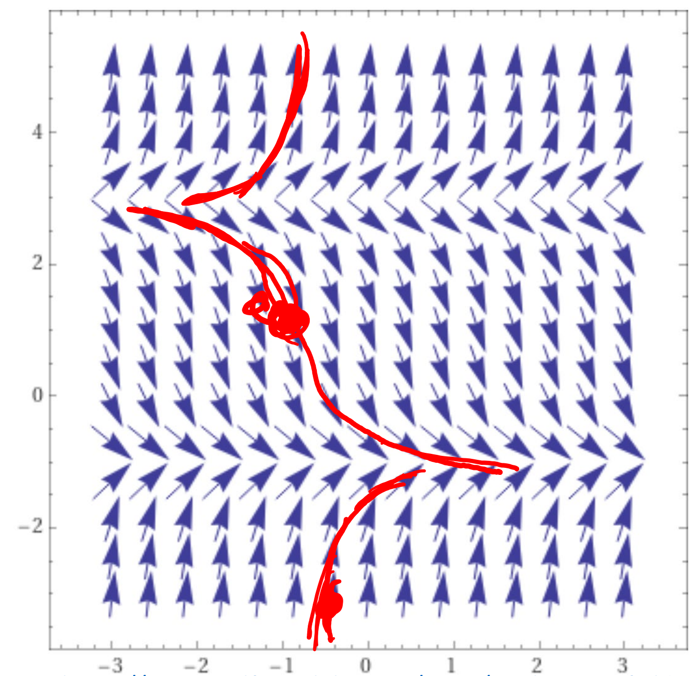
$\boxed{y' = \dfrac{y-3}{y+1}}$



https://www.wolframalpha.com/input/?i=slope%20field

Initial value: A chosen point (t_0, y_0) through which a solution must pass.

I.e., (t_0, y_0) lies on the graph of the solution that satisfies this initial value.



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Initial value problem (IVP): A differential equation where initial value is specified.

An initial value problem can have 0, 1, or multiple equilibrium solutions (finite or infinite).

Long-term behaviour

Suppose a solution $y = f(t)$ to the differential equation $y' = (y-3)(y+1)$ passes thru the point (t_0, y_0).

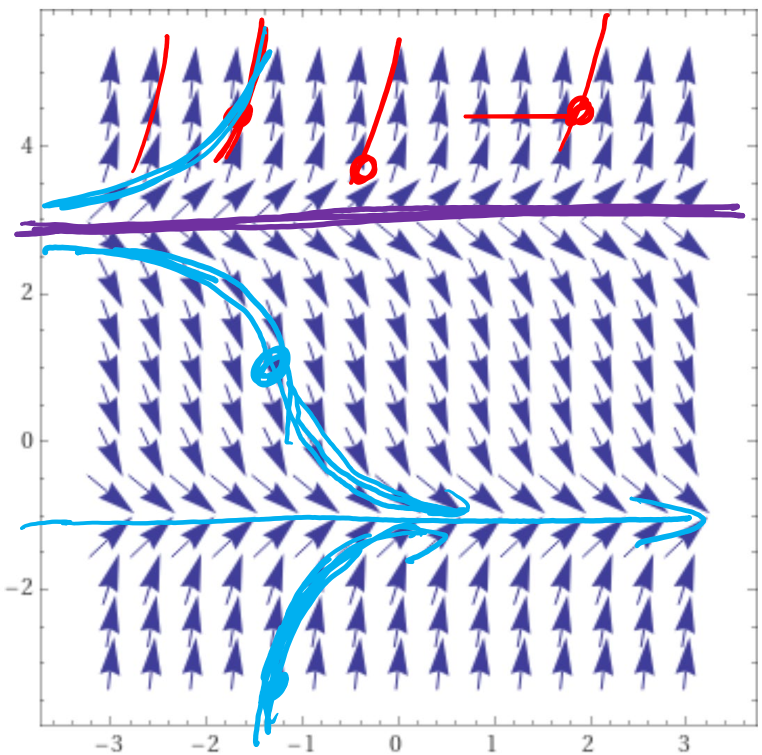
If $y_0 > 3$, then $\lim_{t\to\infty} f(t) = +\infty$

If $y_0 = 3$, then $\lim_{t\to\infty} f(t) = 3$

$f(t) = 3$

If $y_0 < 3$, then $\lim_{t\to\infty} f(t) = -1$

$t \to -\infty$



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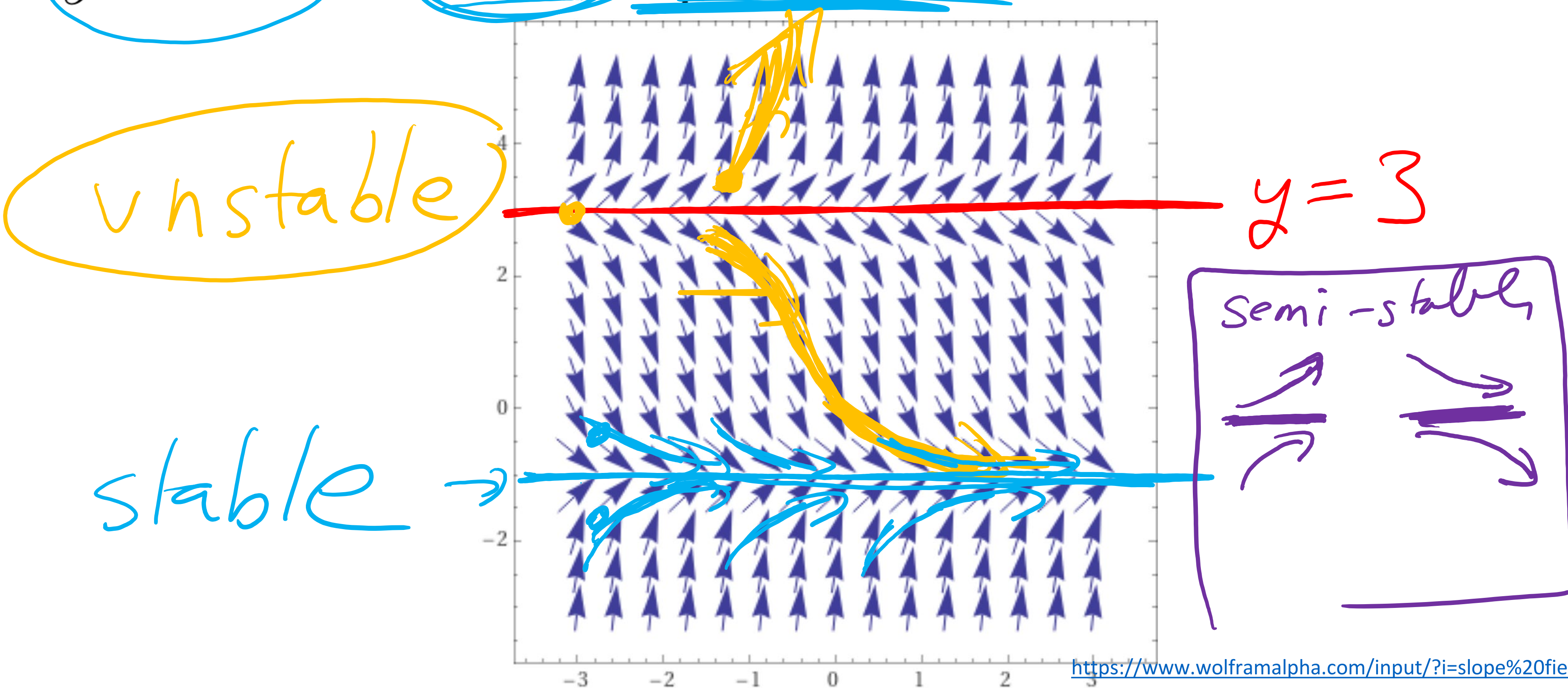
Standard slope field example: $\boxed{y' = (y-3)(y+1)}$

2.5 Preview:

$y = 3$ is an *unstable* equilibrium solution

$y = -1$ is a *stable* equilibrium solution



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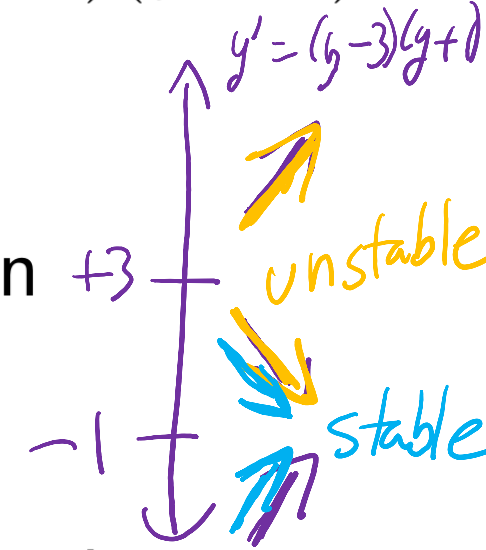


Note: You don't need the slope field graph to determine stability.

Note also that $y' = (y-3)(y+1)$ is *autonomous.*

That is y' depends only on y: $y' = f(y)$

non-autonomous

More complicated slope field example: $y' = -ln(t) + y$

Claim: $\boxed{y' = -ln(t) + y}$ does not have an equilibrium solution.

plug it in to determine if it is a soln

Proof by contradiction:

Suppose $\boxed{y = c}$ is an equilibrium solution. $y = c \Rightarrow y' = 0$

Plugging $y = c$ into DE: $0 = -ln(t) + c$

Thus $c = ln(t)$. But $ln(t)$ not a constant function.

Thus $y' = -ln(t) + y$ does not have an equilibrium solution.

But other non-autom DE can have equil solns

More complicated slope field example: $y' = -ln(t) + y$

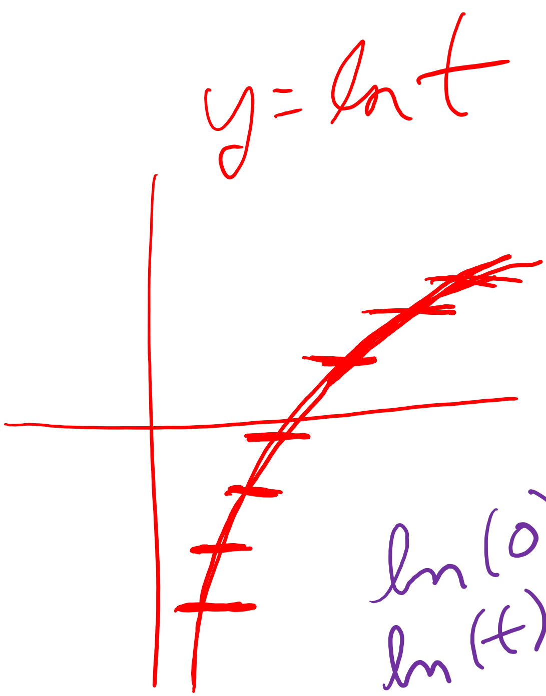
$0 = -\ln t + \ln t$

Note when slope is zero, $0 = -ln(t) + y$

$t = 1 \Rightarrow y' = y$

$\ln(1) = 0$

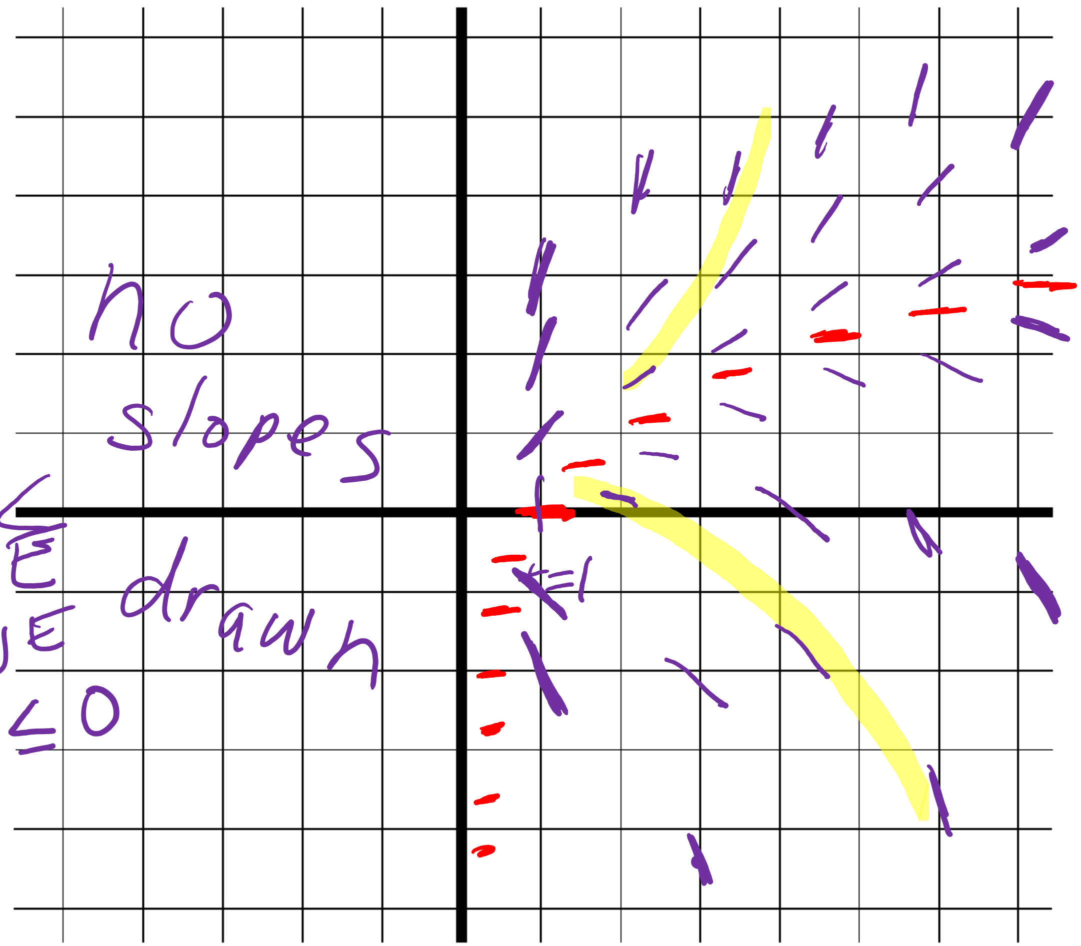
Thus slopes of zero occur along the curve $y = ln(t)$.



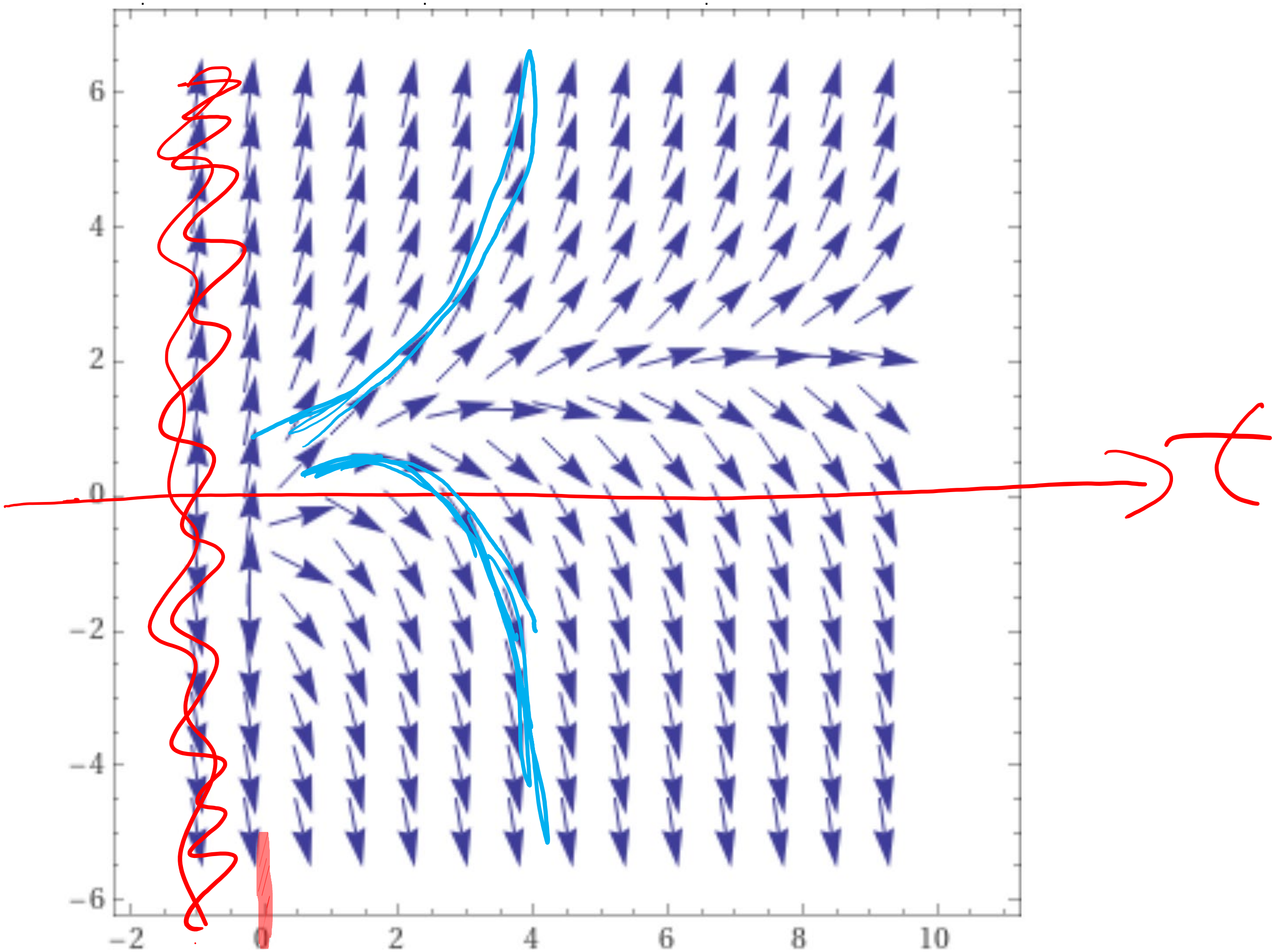
t

$\ln(0)$ DNE

$\ln(t)$ DNE if $t \le 0$



$$y' = -ln(t) + y$$



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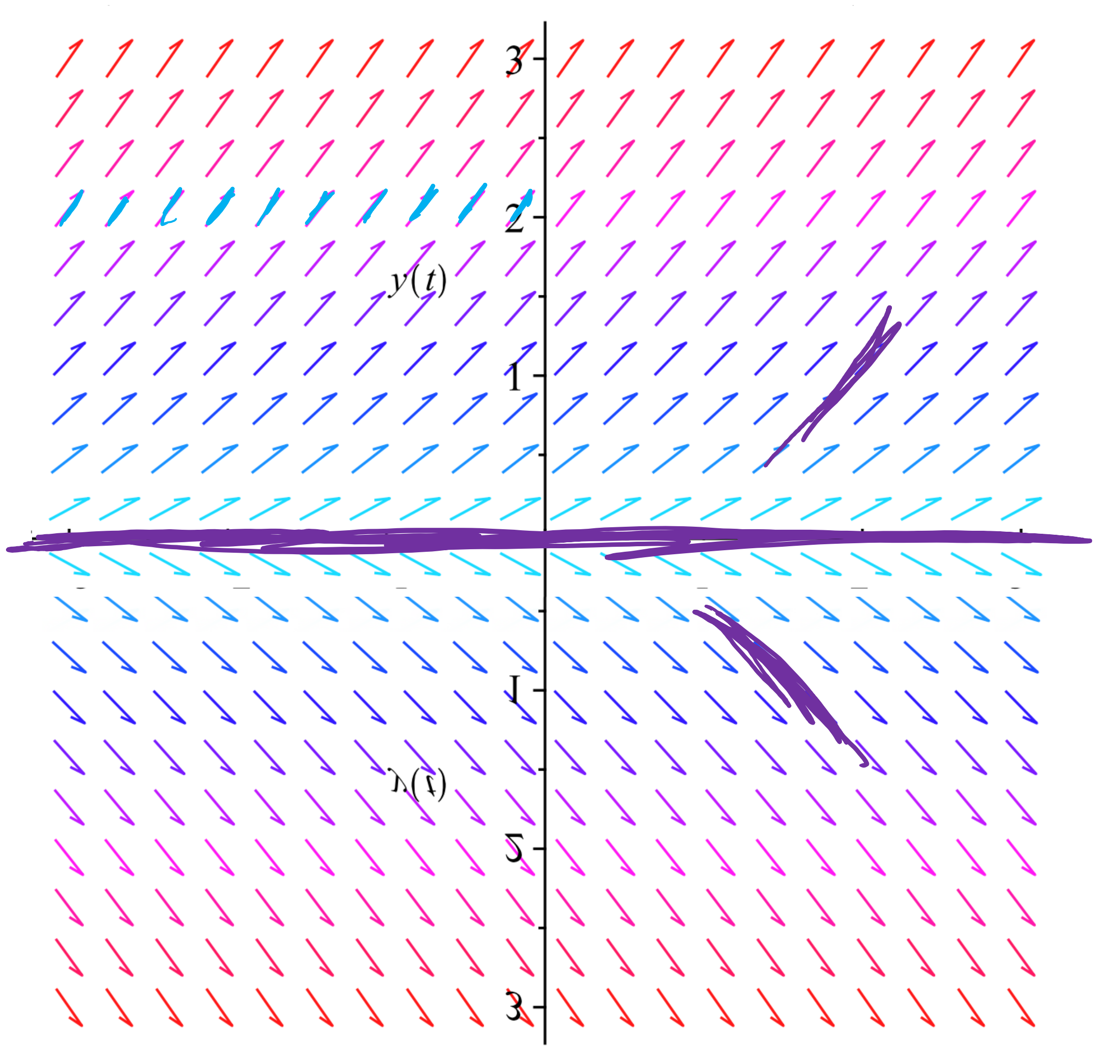
Classic counter-example slope field: $y' = y^{\frac{1}{3}} = f(y)$

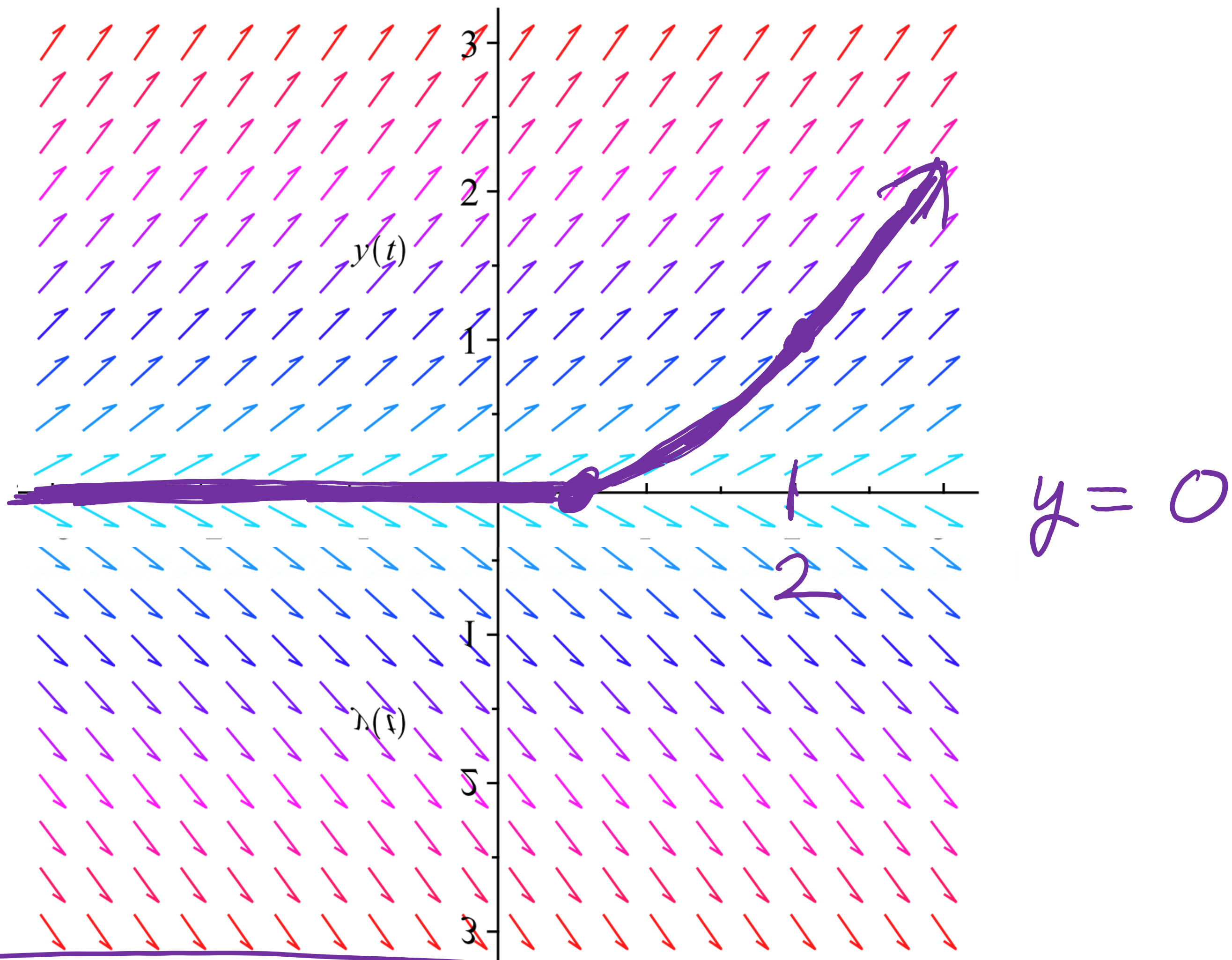
autonomous



$y = 0$ is unstable equil

Is there a horizontal asymptote? NO

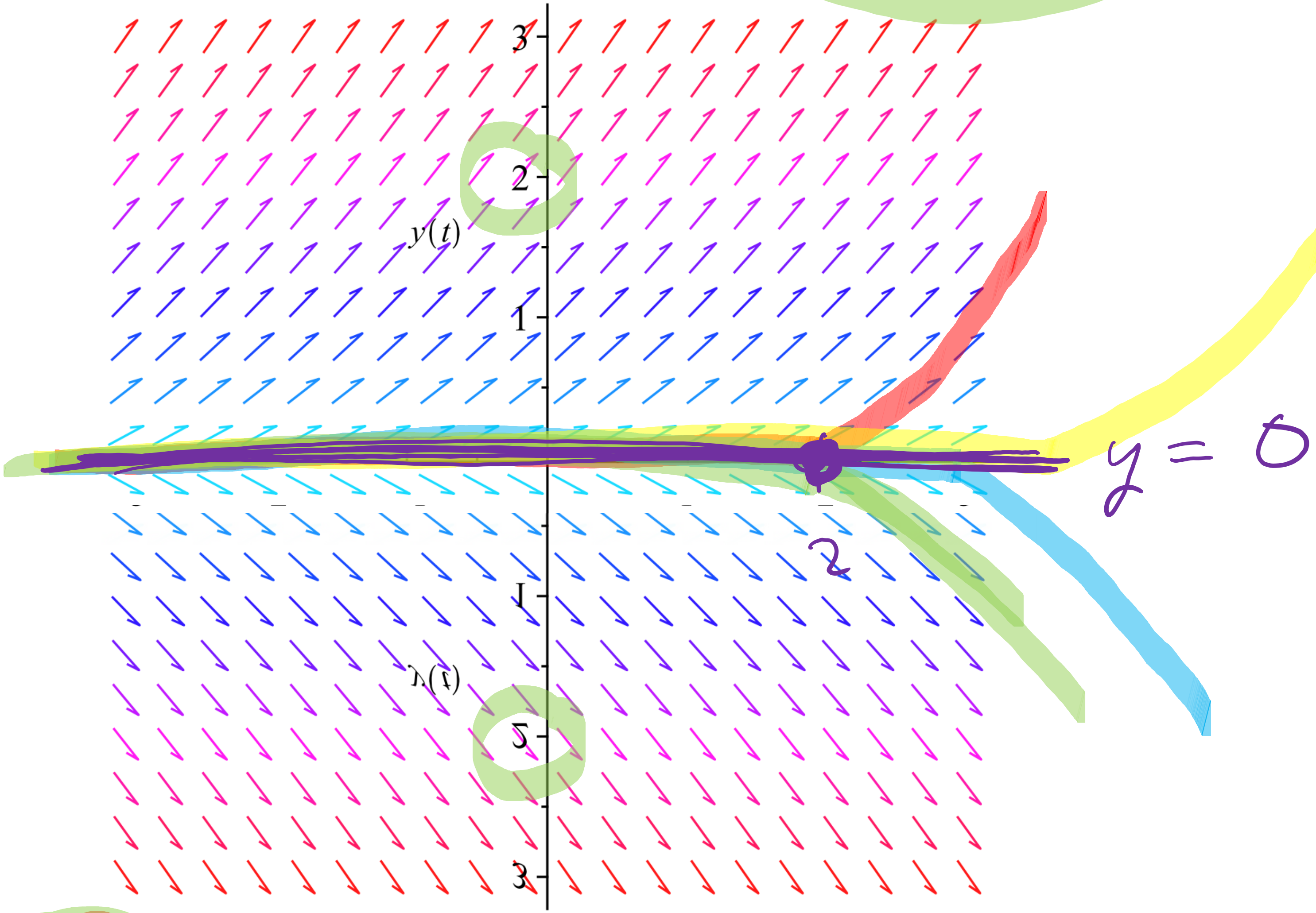
Classic counter-example slope field: $y' = y^{\frac{1}{3}}$



t y

IVP $y' = y^{\frac{1}{3}}$, $y(2) = 1$ has unique solution.

Classic counter-example slope field: $y' = y^{\frac{1}{3}}$



IVP $y' = y^{\frac{1}{3}}$, $y(2) = 0$ has an infinite number of solutions.

> with(DEtools) :

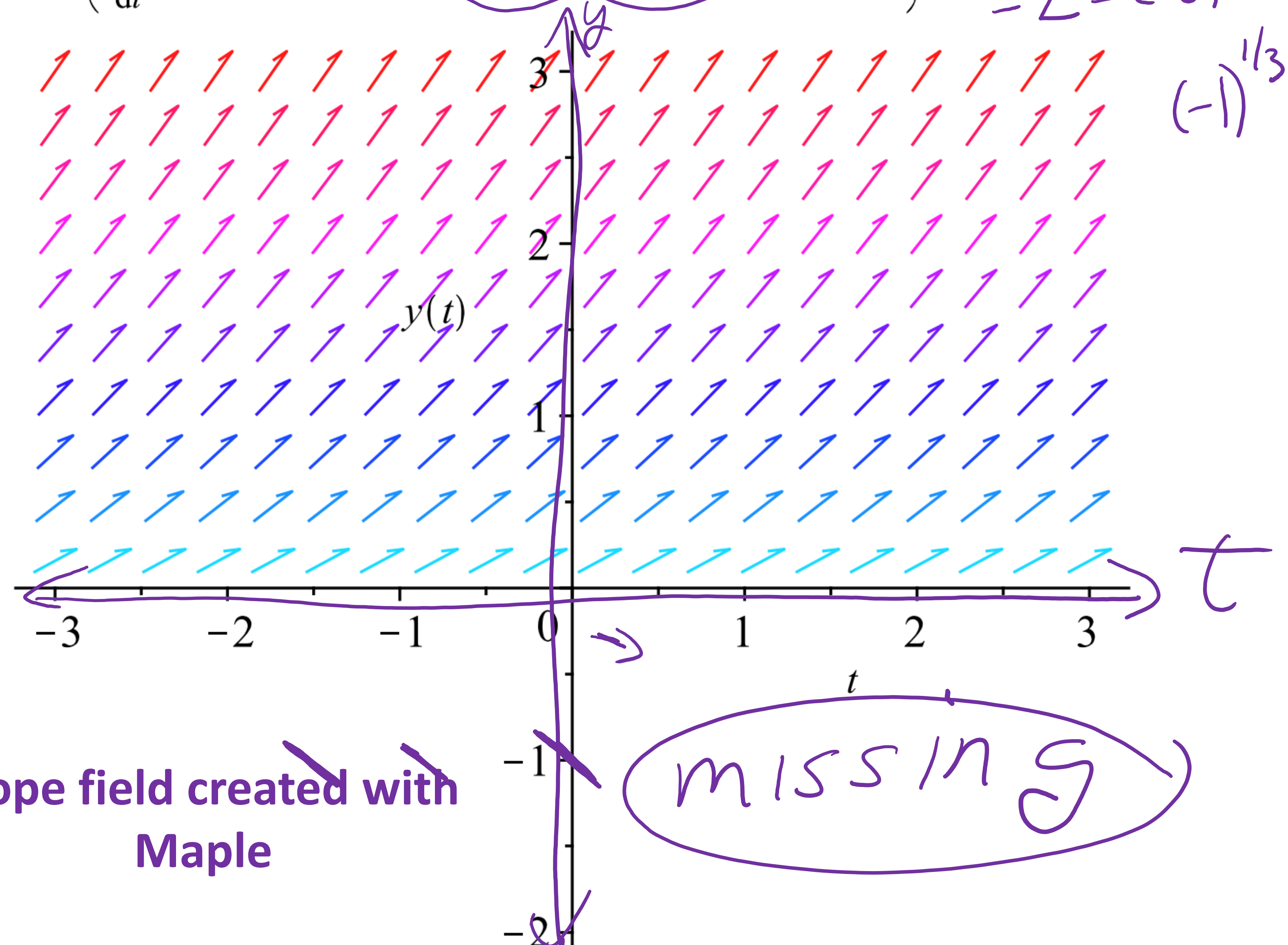
>

> $dfieldplot\left(\frac{d}{dt}\,y(t) = y(t)^{\left(\frac{1}{3}\right)}, y(t), t=-3..3, y=-3..3, color=y(t)\right)$

$y' = y^{1/3}$

$-2 = (-8)^{1/3}$

$(-1)^{1/3}$



Slope field created with Maple

Slope field from wolframalpha.com

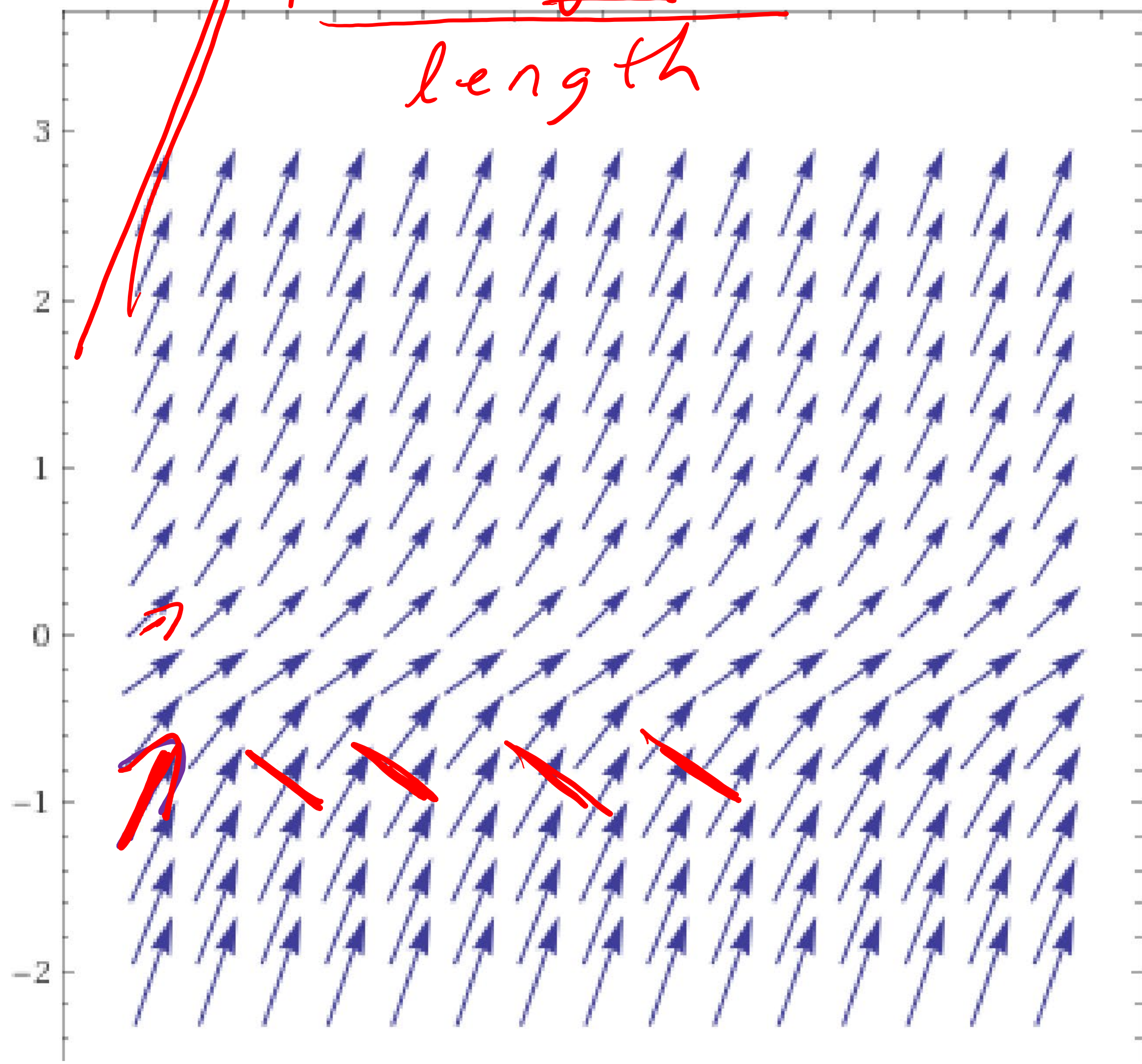
$$\text{VectorPlot}\left[\frac{\{1, \sqrt[3]{y}\}}{\sqrt{y^{2/3}+1}}, \{x, 0, 10\}, \{y, -2, 3\}\right]$$

$$\frac{(1, y^{1/3})}{\text{length}}$$

$$\boxed{y' = y^{1/3}}$$

$y = -1$

$y' = (-1)^{1/3} = -1$



Note: slope is drawn incorrectly. Slope should be negative if *y* < *0*