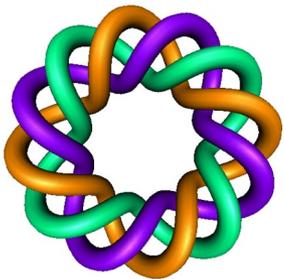


A very elementary introduction to proofs

Part 1

Example: Prove a function is 1:1



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\longleftrightarrow
if and only if

$f : A \rightarrow B$ is 1:1 iff $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Thus to show a function is 1:1, check if

$f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Hypothesis: $f(x_1) = f(x_2)$. Conclusion $x_1 = x_2$.

Hypothesis implies conclusion.

p implies q .

$p \Rightarrow q$.

Note a statement, $p \Rightarrow q$, is true if whenever the hypothesis p holds, then the conclusion q also holds.

To prove that a statement is true:

- (1) Assume the hypothesis holds.
- (2) Prove the conclusion must hold.

Ex: To prove a function is 1:1:

- (1) Assume $f(x_1) = f(x_2)$
- (2) Do some algebra to prove $x_1 = x_2$.

To show a function is 1:1, check if

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2.$$

Goal:
 $x_1 = x_2$

Example: Show $f(x) = \ln(x)$ is 1:1

Proof: Suppose $f(x_1) = f(x_2)$

$$\ln(x_1) = \ln(x_2)$$

$$e^{\ln(x_1)} = e^{\ln(x_2)} \Rightarrow x_1 = x_2 \quad \square$$

To show a function is 1:1, check if

$$\underline{f(x_1) = f(x_2)} \text{ implies } x_1 = x_2.$$

Example: Show $f(x) = \underline{\ln(x)}$ is 1:1

Proof:

$$\underline{\ln(x_1) = \ln(x_2)} \Rightarrow e^{\ln(x_1)} = e^{\ln(x_2)} \Rightarrow x_1 = x_2.$$

To show a function is 1:1, check if

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2.$$

Example: Show $f(x) = \ln(x)$ is 1:1

Proof: **Suppose the hypothesis:**

Suppose $f(x_1) = f(x_2)$

Know this
you can work
with this

That is, suppose $\ln(x_1) = \ln(x_2)$.

re word
(be more
specific)

Prove the conclusion holds:

Claim: $x_1 = x_2$.

$$\ln(x_1) = \ln(x_2) \Rightarrow e^{\ln(x_1)} = e^{\ln(x_2)} \Rightarrow x_1 = x_2.$$

Some notation:

\forall = for all

\exists = there exists

$[p \Rightarrow q]$ is equivalent to $[\forall p, q \text{ holds}]$.

$p \Rightarrow q$

$\forall p, q$

That is, for everything satisfying the hypothesis p , the conclusion q must hold.

$f : A \rightarrow B$ is 1:1 iff

$f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$f : A \rightarrow B$ is 1:1 iff

$\forall x_1$ and $\forall x_2$ such that $f(x_1) = f(x_2)$,

we have $x_1 = x_2$.