In Class Quizzes Part 1

- In Class Quizzes prior to Midterm 1 on September 23 (ICQs between 8/24 and 9/21), worth 14 points total.

- Note that the highest grade on this assignment is 100%.

- If your score on this assignment is less than 100%, you can still bring your score up to 100%.

In Class Quizzes Part 2 (10/7 - Week 13)

Bonus points: All quizzes 10/7 – 10/21 , All quizzes ? - ?, Final exam all quizzes 8/24 - ??, Posting on ICON discussion page, chats from last Wednesday,

Talk about math and game theory in political science

The AWM (Association for Women in Mathematics) student chapter is hosting a talk this Thursday, October 22 from 3:30 to 4:30 via Zoom (see link below). Dr. Elizabeth Menninga, Assistant Professor in the Department of Political Science, will be talking about how her studies in math led her to research in political science.

This talk will be geared toward undergraduates.

Zoom Meeting Details:
https://uiowa.zoom.us/j/98706452660?pwd=c3ZsUUkzV2FWTW1uV3FoOXBjOG9tQT09

Meeting ID: 987 0645 2660
Passcode: 119820

Linear combination:

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y$$

Linear DE:

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y = g(t)$$

Homogeneous DE:

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y = 0$$

IVP:

$$y^{(n)} + p_1(t)y^{(n-1)} + ... + p_{n-1}(t)y' + p_n(t)y = g(t),$$

$$y(t_0) = y_0,\ y'(t_0) = y_1, ..., y^{(n-1)}(t_0) = y_{n-1}$$

Suppose $a_0 y^{(n)} + a_1 y^{(n-1)} + ... + a_{n-1} y' + a_n y = g(t)$

has solution $y = c_1\phi_1(t) + ... + c_n\phi_n(t) + \psi(t)$

Characteristic polynomial

Fundamental set of solutions

Wronskian

Wronskian

Wronskian

7.1: Transforming an n^{th} order linear DE into a system of *n* first order linear DEs.

Ex: $y'''' - 5y'' + 6y = \sin(t)$

7.4 - 7.6, 9.1

Solve the homogeneous linear DE: $\mathbf{x}' - A\mathbf{x} = \mathbf{0}$

Solve the homogeneous linear DE: $\mathbf{x}' - A\mathbf{x} = \mathbf{0}$

$\mathbf{x}' = A\mathbf{x}$ Guess $x = \mathbf{v}e^{rt}$. Plug in to find $\mathbf{v}$ and r:

$$[\mathbf{v}e^{rt}]' = A\mathbf{v}e^{rt} \quad \text{implies} \quad r\mathbf{v}e^{rt} = A\mathbf{v}e^{rt} \quad \text{implies} \quad r\mathbf{v} = A\mathbf{v}.$$

Thus $\mathbf{v}$ is an eigenvector with eigenvalue r.

Note since the equation is homogeneous and linear,

linear combinations of solutions are also solutions:

Suppose $\mathbf{x} = \mathbf{f_1}(t)$ and $\mathbf{x} = \mathbf{f_2}(t)$ are solutions to $\mathbf{x}' = A\mathbf{x}$.

Then $\mathbf{f_1}' = A\mathbf{f_1}$ and $\mathbf{f_2}' = A\mathbf{f_2}$

Thus $[c_1\mathbf{f_1} + c_2\mathbf{f_2}]' = c_1\mathbf{f_1}' + c_2\mathbf{f_2}' = c_1 A\mathbf{f_1} + c_2 A\mathbf{f_2} = A(c_1\mathbf{f_1} + c_2\mathbf{f_2})$.

Suppose an object moves in the 2D plane (the x_1, x_2 plane) so that it is at the point $(x_1(t), x_2(t))$ at time t. Suppose the object's velocity is given by

$$x_1'(t) = 4x_1 + x_2,$$
$$x_2'(t) = 5x_1$$

Or in matrix form $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Guess $x = \mathbf{v}e^{rt}$

To solve, find eigenvalues and corresponding eigenvectors:

$$\begin{vmatrix} 4-r & 1 \\ 5 & -r \end{vmatrix} = (4-r)(-r) - 5 = r^2 - 4r - 5 = (r-5)(r+1).$$

Thus $r = -1, 5$ are eigenvalues.

Eigenvectors associated to eigenvalue $r = -1$: $\begin{pmatrix} 5 & 1 \\ 5 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{5} \\ 0 & 0 \end{pmatrix}$

Thus x_2 is free and $x_1 + \frac{1}{5}x_2 = 0$

Hence the eigenspace corresponding to $r = -1$ is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{5}x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -\frac{1}{5} \\ 1 \end{pmatrix}$$

Thus $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ is an eigenvector with eigenvalue $r = -1$

Hence $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} e^{-t}$ is a solution.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Guess $x = \mathbf{v}e^{rt}$

To solve, find eigenvalues and corresponding eigenvectors:

$$\begin{vmatrix} 4-r & 1 \\ 5 & -r \end{vmatrix} = (4-r)(-r) - 5 = r^2 - 4r - 5 = (r-5)(r+1).$$

Thus $r = -1, 5$ are eigenvalues.

E. vectors associated to e. value $r = 5$: $\begin{pmatrix} -1 & 1 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Thus $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector with eigenvalue $r = 5$ since it is a nonzero solution to the above equation.

Hence $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$ is also a solution.

Note since the equation is homogeneous and linear,

linear combinations of solutions are also solutions:

Suppose $\mathbf{x} = \mathbf{f_1}(t)$ and $\mathbf{x} = \mathbf{f_2}(t)$ are solutions to $\mathbf{x}' = A\mathbf{x}$.

Then $\mathbf{f_1}' = A\mathbf{f_1}$ and $\mathbf{f_2}' = A\mathbf{f_2}$

Thus $[c_1\mathbf{f_1} + c_2\mathbf{f_2}]' = c_1\mathbf{f_1}' + c_2\mathbf{f_2}' = c_1A\mathbf{f_1} + c_2A\mathbf{f_2} = A(c_1\mathbf{f_1} + c_2\mathbf{f_2})$.

Hence the general solutions is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 5 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$

Or in non-matrix form: $x_1(t) = -c_1e^{-t} + c_2e^{5t}$
$x_2(t) = 5c_1e^{-t} + c_2e^{5t}$