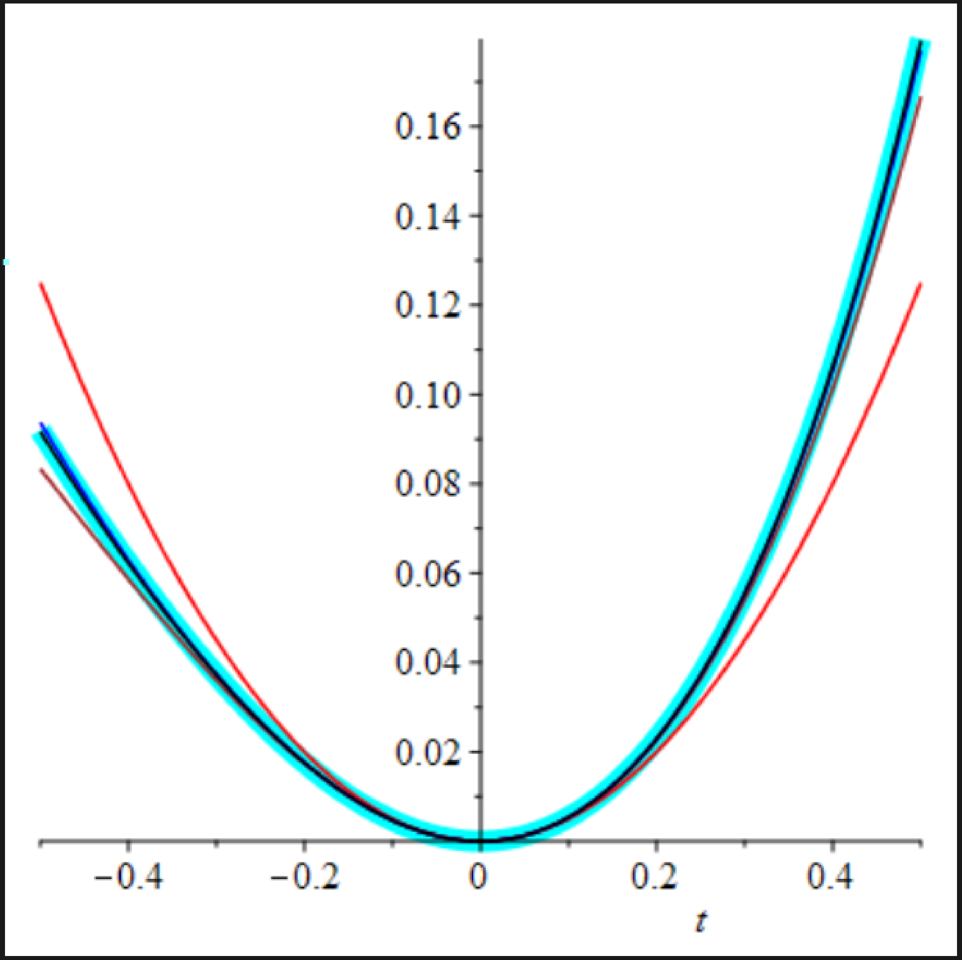
$e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$ and thus $e^{bt} = \sum_{k=0}^{\infty} \frac{b^k t^k}{k!}$ for t near 0.

$$\phi_n(t) = \sum_{k=2}^{n} \frac{2^{k-2}}{k!} t^k$$

2.8: Approximating soln to IVP using seq of fns.

The solution $y = \phi(t)$ is the thick cyan curve.



$$\phi_0(t) = 0$$

$$\phi_1(t) = \frac{t^2}{2}$$

$$\phi_2(t) = \frac{t^2}{2} + \frac{t^3}{3}$$

$$\phi_3(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6}$$

$$\phi_4(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6} + \frac{t^5}{15}$$

Section 2.5: Autonomous equations:

$$y' = f(y)$$

Note the change in y only depends on y

Compare to calculus 1 problems: $y' = f(t)$

Example: Exponential Growth/Decay

Example: population growth/radioactive decay

Change in y is proportional to y

$$y' = ry, \quad y(0) = y_0 \text{ implies } y = y_0 e^{rt}$$

Equilibrium solution:

$r > 0$ $\qquad\qquad$ $r < 0$

Example: $y' = ry, \quad y(0) = y_0$ implies $y = y_0 e^{rt}$

Equilibrium solution: $y = 0$

$r > 0$ $\qquad$ y vs $f(y)$: $\qquad\qquad$ slope field:

Determine long-term behaviour:

$\lim_{t \to \infty} y(t) =$ $\qquad\qquad$ $\lim_{t \to -\infty} y(t) =$

Example: $y' = ry, \quad y(0) = y_0$ implies $y = y_0 e^{rt}$

Equilibrium solution: $y = 0$

$r < 0$ $\quad y$ vs $f(y)$: $\qquad$ slope field:

Determine long-term behaviour:

$\lim_{t \to \infty} y(t) =$ $\qquad$ $\lim_{t \to -\infty} y(t) =$

Example: population growth

Change in y is proportional to y:

$$y' = ry \text{ where } r \text{ is the growth rate.}$$

Example: Logistic growth:

$$y' = h(y)y$$

The growth rate $h(y)$ depends on the population y.

Example: Logistic growth: $y' = h(y)y$

Example: $y' = r(1 - \frac{y}{K})y$

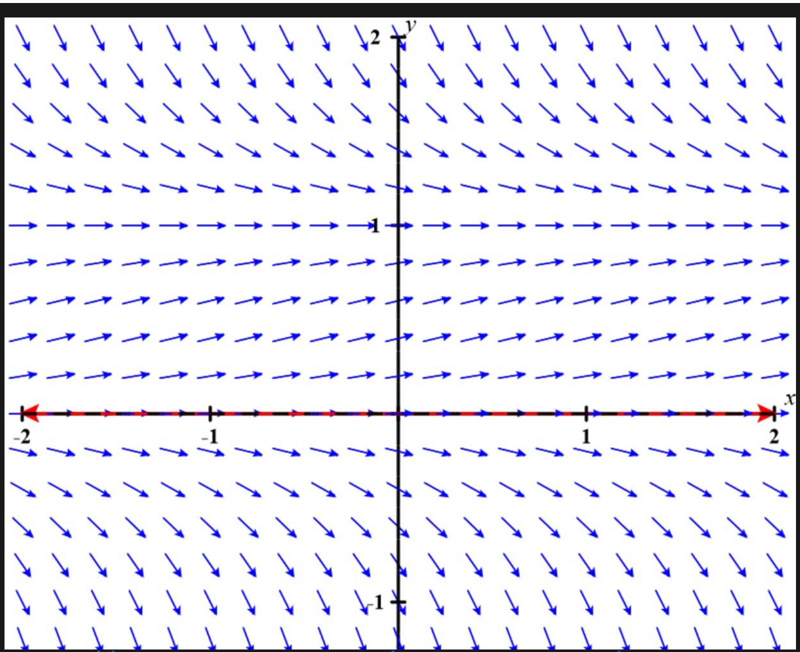
Equilibrium solutions:

$r > 0$ $\quad$ y vs $f(y)$: $\quad$ slope field:

Example: Logistic growth: $y' = h(y)y$

Example: $y' = r(1 - \frac{y}{K})y$

Solution: $y = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$



https://c3d.libretexts.org/DirectionField/index.html

Determine long-term behaviour:

$\lim_{t\to\infty} y(t) =$

$\lim_{t\to-\infty} y(t) =$

Equilibrium solution

Asymptotically stable:

Asymptotically unstable:

Asymptotically semi-stable:

Section 2.5 Autonomous equations: $y' = f(y)$

If given either differential equation $y' = f(y)$

OR direction field:

- Find equilibrium solutions and determine if stable, unstable, semi-stable.
- Understand what the above means.

Determine long-term behaviour:

$$\lim_{t\to\infty} y(t) =$$

$$\lim_{t\to-\infty} y(t) =$$

A function f is linear if $f(a\mathbf{x}+b\mathbf{y}) = af(\mathbf{x})+bf(\mathbf{y})$

Or equivalently f is linear if

1.) $f(a\mathbf{x}) = af(\mathbf{x})$ and

2.) $f(\mathbf{x}+\mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y})$

Example: Prove $y = ln(t)$ is not a linear function.