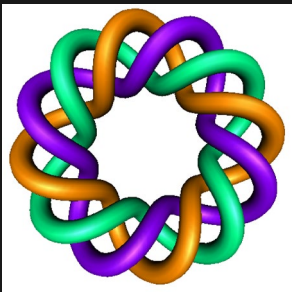


Solving Bernoulli's equation

$$y' + p(t)y = g(t)y^n$$



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2.4 Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n$$

- ▶ If $n = 0$, then $y' + p(t)y = g(t)$ is linear.
- ▶ If $n = 1$, then $y' + p(t)y = g(t)y$ is separable.

When $n \neq 0, 1$, solve Bernoulli's by changing it to a linear equation.

How do you change a
problem that you do
not know how to solve
into a problem that
you do know how to
solve?

The algorithm is NOT
as important as the
idea behind the
algorithm.

Goal: Create linear equation, $w' + P(t)w = G(t)$

2.4 Solve Bernoulli's equation when $n \neq 0, 1$ by changing it to a linear equation

$$y' + p(t)y = g(t)y^n$$

Goal: Create linear equation, $w' + P(t)w = G(t)$

2.4 Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

and noting that $v' = (1 - n)y^{-n}y'$

Goal: Create linear equation, $w' + P(t)w = G(t)$

Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

Goal: Create linear equation, $w' + P(t)w = G(t)$

$$\text{Solve } ty' + 2t^{-2}y = 2t^{-2}y^5$$

$$ty^{-5}y' + 2t^{-2}y^{-4} = 2t^{-2}$$

Let $v = y^{-4}$. Thus $v' = -4y^{-5}y'$

$$-4ty^{-5}y' - 8t^{-2}y^{-4} = -8t^{-2}$$

$$tv' - 8t^{-2}v = -8t^{-2}$$

We now have a linear DE that we can solve for v

$$tv' - 8t^{-2}v = -8t^{-2}$$

Solve linear DE for v :

Make coefficient of $v' = 1$

$$v' - 8t^{-3}v = -8t^{-3}$$

Find integrating factor:

An antiderivative of $-8t^{-3}$ is $4t^{-2}$

Thus integrating factor is $u(t) = e^{\int -8t^{-3}dt} = e^{4t^{-2}}$.

Multiply equation by $e^{4t^{-2}}$

$$e^{4t^{-2}}v' - 8t^{-3}e^{4t^{-2}}v = -8t^{-3}e^{4t^{-2}}$$

$$e^{4t^{-2}} v' - 8t^{-3} e^{4t^{-2}} v = -8t^{-3} e^{4t^{-2}}$$

$$(e^{4t^{-2}} v)' = -8t^{-3} e^{4t^{-2}} \text{ by PRODUCT rule.}$$

$$\int (e^{4t^{-2}} v)' dt = -8 \int t^{-3} e^{4t^{-2}} dt$$

$$e^{4t^{-2}} v = -8 \int t^{-3} e^{4t^{-2}} dt.$$

Integrate RHS using substitution:

$$\text{Let } u = 4t^{-2}. \text{ Then } du = -8t^{-3} dt$$

$$e^{4t^{-2}} v = \int e^u du = e^u + C$$

$$e^{4t^{-2}} v = e^{4t^{-2}} + C$$

$$v = 1 + C e^{-4t^{-2}}$$

$$v = 1 + Ce^{-4t^{-2}}$$

$$\text{Solve } ty' + 2t^{-2}y = 2t^{-2}y^5$$

$$\text{Recall } v = y^{-4}:$$

$$y^{-4} = 1 + Ce^{-4t^{-2}} \text{ implies } y = \pm(1 + Ce^{-4t^{-2}})^{-\frac{1}{4}}$$

$$\text{General solution: } y = \pm(1 + Ce^{-4t^{-2}})^{-\frac{1}{4}}$$

$$\text{OR } y = 0$$