

Product Rule: $(fg)' = fg' + f'g$

Application 1: Integration by parts

$$\int fg' dt = \int (fg)' dt - \int f'g dt = fg - \int f'g dt$$

Application 2: (2.1) Solve 1st order linear DE

MOTIVATING

EX 1 :

product rule

$$t^2 y' + 2ty = \cos(t)$$

$$\int [t^2 y]' dt = \int \cos(t) dt$$

$$t^2 y = \sin(t) + C$$

$$\Rightarrow y = \frac{\sin t}{t^2} + \frac{C}{t^2}$$

EX 2: $ty' + 9y = \frac{e^{2t}}{t^5}$

Create product rule by using
 integrating factor $u(t) = e^{\int p(t) dt}$
 for linear DE $1y' + p(t)y = g(t)$

$\div t:$ $1y' + \frac{9}{t}y = \frac{e^{2t}}{t^5}$

$$1y' + \frac{9}{t}y = \frac{e^{2t}}{t^5}$$

Integrating factor

$$u(t) = e^{\int p(t) dt} = e^{\int \frac{9}{t} dt} = e^{9 \int \frac{1}{t} dt}$$

$$= e^{9 \ln |t|} = e^{\ln |t|^9} = |t|^9$$

Choose $u(t) = t^9$

$$t^9 [1y' + \frac{9}{t}y] = \left[\frac{e^{2t}}{t^5} \right] t^9$$

$$t^9 y' + 9t^8 y = t^3 e^{2t}$$

$$\int [t^9 y]' dt = \int t^3 e^{2t} dt$$

$$t^9 y = \int t^3 e^{2t} dt$$

$$t^9 y' = \int t^3 e^{2t} dt$$

$$= \frac{t^3 e^{2t}}{2} - \int 3t^2 \frac{e^{2t}}{2} dt$$

Integration by parts

Let $u = t^3$ $dv = e^{2t}$

$du = 3t^2$ $v = e^{2t}/2$

$d^2u = 6t$ $\int v = e^{2t}/4$

$d^3u = 6$ $\int \int v = e^{2t}/8$

$d^4u = 0$ $\int \int \int v = e^{2t}/16$

$\int u dv = uv - \int v du$

$$t^9 y' = \frac{t^3 e^{2t}}{2} - \frac{3t^2 e^{2t}}{4} + \frac{6te^{2t}}{8} - \frac{6e^{2t}}{16} + \int 0 dt$$

$$= \frac{t^3 e^{2t}}{2} - \frac{3t^2 e^{2t}}{4} + \frac{3te^{2t}}{4} - \frac{3e^{2t}}{8} + C$$

$$y' = \frac{e^{2t}}{t^6} - \frac{3e^{2t}}{4t^7} + \frac{3e^{2t}}{4t^8} - \frac{3e^{2t}}{8t^9} + \frac{C}{t^9}$$

↑
don't forget $\frac{C}{t^9}$