

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. $e^{at}$	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 31
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Ex. 7
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s >  a $	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s >  a $	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 25
16. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	$e^{-cs}$	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	Sec. 6.2; Cor. 6.2.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 29

The algebra techniques that you will use in ch 6 include  
partial fractions,  
completing the square,  
adding 0, and  
multiplying by 1.

1. Look at denominator

i.) Can you factor denominator over the reals? If so, factor and use partial fractions.

ii.) Does your denominator equal one of the following?  $s^n$ ,  $s - a$ ,  $s^2 \pm a^2$ , If so, use the appropriate formula.

iii.) Do you need to complete the square? Example:

$$\begin{aligned}10s^2 + 60s + 91 &= 10(s^2 + 6s) + 91 \\ &= 10(s^2 + 6 + 9 - 9) + 91 \\ &= 10(s^2 + 6 + 9) - 90 + 91 \\ &= 10(s + 3)^2 + 1\end{aligned}$$

2. Look at the numerator

i.) Do you need  $s - a$ ? Try adding 0. For example to make  $s + \frac{3}{2}$  appear in  $5s + 21$ :

$$5s + 21 = 5\left(s + \frac{3}{2}\right) - \frac{15}{2} + 21 = 5\left(s + \frac{3}{2}\right) + \frac{27}{2}$$

ii) Do you need  $b$ ? Try multiplying by 1. For example, if you need  $\sqrt{\frac{7}{4}}$ , but you have  $\frac{27}{2}$ :  $\frac{27}{2} = \frac{27}{2} \sqrt{\frac{4}{7}} \sqrt{\frac{7}{4}}$