

To solve 1st order linear DE: $1y' + p(t)y = g(t)$

Create product rule by using integrating factor: $u(t) = e^{\int p(t)dt}$

Ex 1: $ty' + 9y = \frac{e^{2t}}{t^5} \Rightarrow 1y' + \frac{9}{t}y = \frac{e^{2t}}{t^6}$

Create product rule by using integrating factor:

$u(t) = e^{\int p(t)dt} = e^{\int \frac{9}{t}dt} = e^{9\ln(t)} = e^{\ln(t)^9} = t^9$

$t^9(y' + \frac{9}{t}y) = t^9(\frac{e^{2t}}{t^6})$

$t^9y' + 9t^8y = t^3e^{2t}$

$(t^9y)' = t^3e^{2t} \Rightarrow \int (t^9y)' = \int t^3e^{2t}dt$

$t^9y = \int t^3e^{2t}dt = t^3e^{2t} - \int 3t^2\frac{e^{2t}}{2}dt$

$t^9y = t^3\frac{e^{2t}}{2} - \frac{3t^2e^{2t}}{4} + \frac{6te^{2t}}{8} - \frac{6e^{2t}}{16} + C$

General solution: $y = \frac{e^{2t}}{2t^6} - \frac{3e^{2t}}{4t^7} + \frac{3e^{2t}}{4t^8} - \frac{3e^{2t}}{8t^9} + \frac{C}{t^9}$

Use integration by parts on RHS

$u = t^3 \quad dv = e^{2t}$

$du = 3t^2 \quad v = \frac{e^{2t}}{2}$

$d^2u = 6t \quad \int v = \frac{e^{2t}}{4}$

$d^3u = 6 \quad \int \int v = \frac{e^{2t}}{8}$

$d^3u = 0 \quad \int \int \int v = \frac{e^{2t}}{16}$

linear comb of y and y'

Solve for y by dividing both sides by integrating factor

C
integrating factor

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Integrating factor