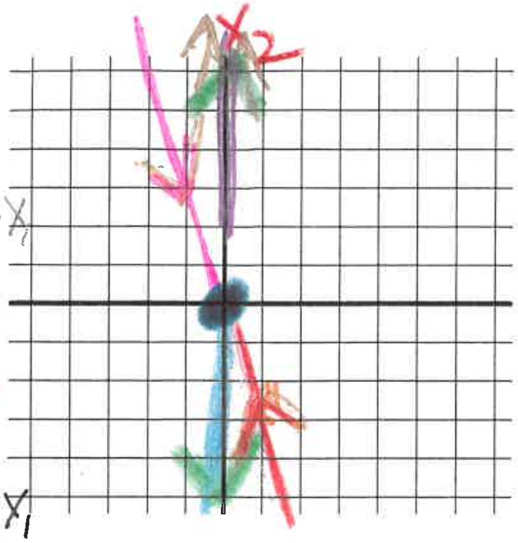
7.5 Two real eigenvectors: Graph several trajectories for the following systems of equations:

Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$



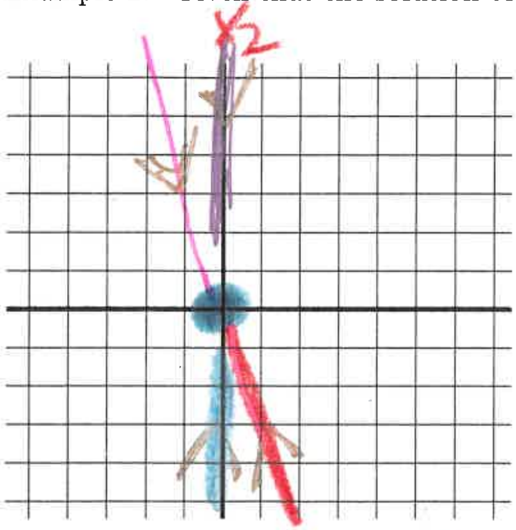
The equilibrium solution for this system of equations is

This equilibrium solution is (choose one):

asymptotically stable or unstable

Classify this critical point's type:

Example 2: Given that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$



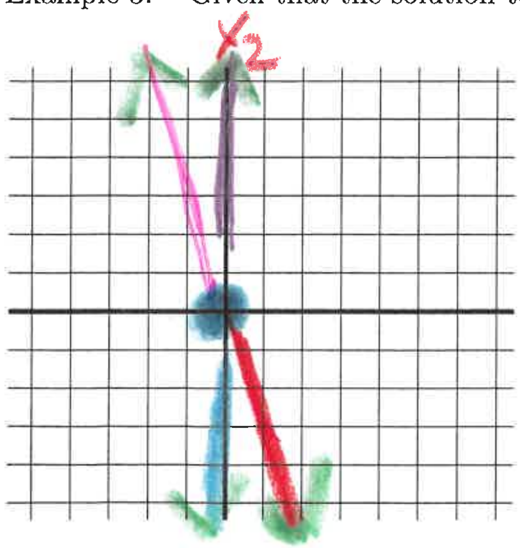
The equilibrium solution for this system of equations is

This equilibrium solution is (choose one):

asymptotically stable or unstable

Classify this critical point's type:

Example 3: Given that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$



The equilibrium solution for this system of equations is

This equilibrium solution is (choose one):

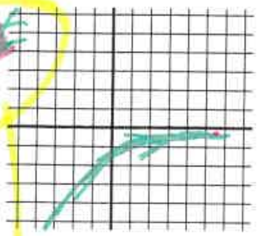
asymptotically stable or unstable

Classify this critical point's type:

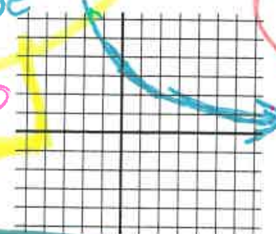
Example 1: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

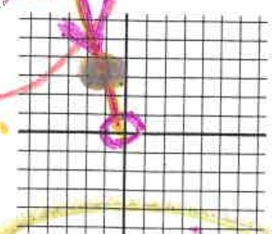
t, x_1-plane



t, x_2-plane

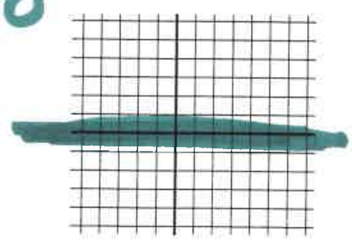


x_1, x_2-plane

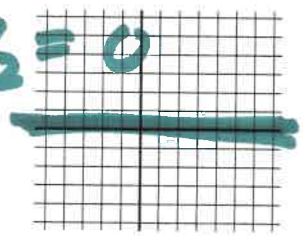


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

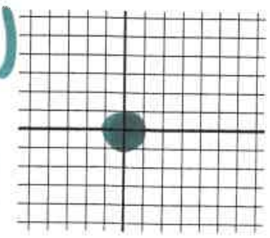
t, x_1-plane



t, x_2-plane



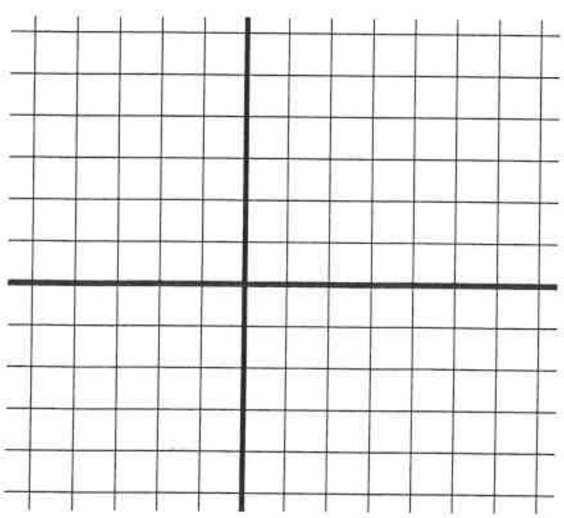
x_1, x_2-plane



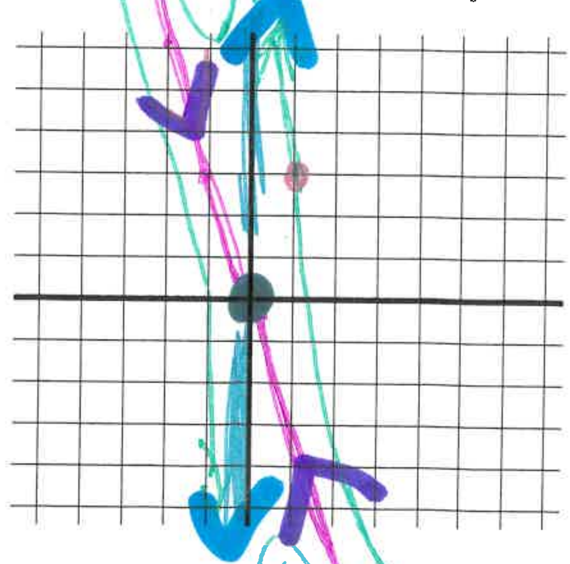
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ______________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Semi-generic ex: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

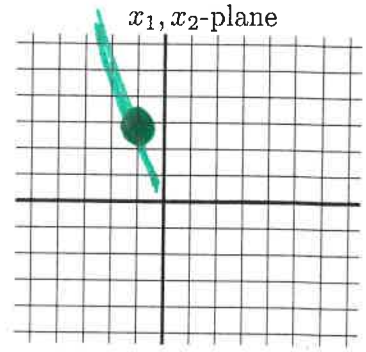
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ implies $c_1 = 1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = -e^{r_1 t} < 0$ and $x_2 = 3e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -e∧(-2t), 3*e∧(-2t))

x_1, x_2-plane



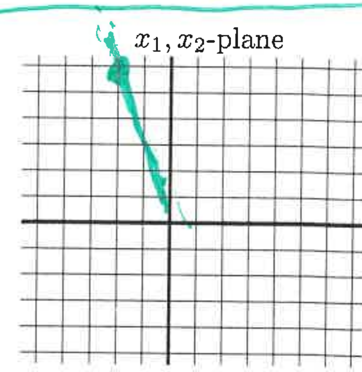
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ implies $c_1 = 2, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = -2e^{r_1 t} < 0$ and $x_2 = 6e^{r_1 t} > 0$

and $\frac{x_2}{x_1} = \frac{6e^{r_1 t}}{-2e^{r_1 t}} = \frac{3}{-1}$. Thus $x_2 = \frac{3}{-1} x_1$.

https://www.geogebra.org/3d (t, -2*e∧(-2t), 6*e∧(-2t))

x_1, x_2-plane



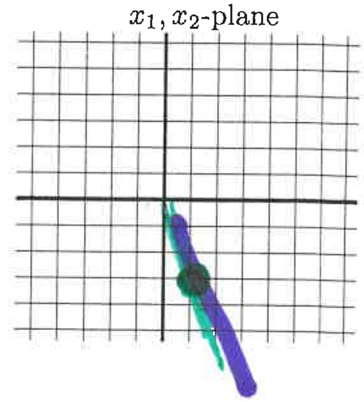
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ implies $c_1 = -1, c_2 = 0$. Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{r_1 t}$

Hence $x_1 = e^{r_1 t} > 0$ and $x_2 = -3e^{r_1 t} < 0$

and $\frac{x_2}{x_1} = \frac{3e^{r_1 t}}{-e^{r_1 t}} = \frac{-3}{1}$. Thus $x_2 = \frac{-3}{1} x_1$.

https://www.geogebra.org/3d (t, e∧(-2t), -3*e∧(-2t))

x_1, x_2-plane



Semi-generic ex: Given that the solution to to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{r_2 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ implies $c_1 = 0, c_2 = 1$.

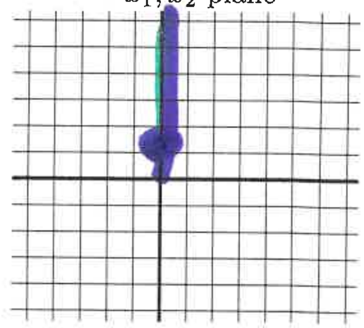
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = e^{r_2 t} > 0$

and $\frac{x_2}{x_1} = \frac{1e^{r_2 t}}{0e^{r_2 t}} = \frac{1}{0}$. Thus $x_2 = \frac{1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, e∧(5t))

x_1, x_2-plane



IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ implies $c_1 = 0, c_2 = -1$.

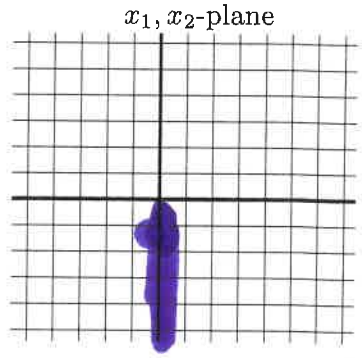
Thus IVP soln: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{r_2 t}$

Hence $x_1 = 0$ and $x_2 = -e^{r_2 t} < 0$

and $\frac{x_2}{x_1} = \frac{-1e^{r_2 t}}{0e^{r_2 t}} = \frac{-1}{0}$. Thus $x_2 = \frac{-1}{0} x_1$.

https://www.geogebra.org/3d (t, 0, -e∧(5t))

x_1, x_2-plane



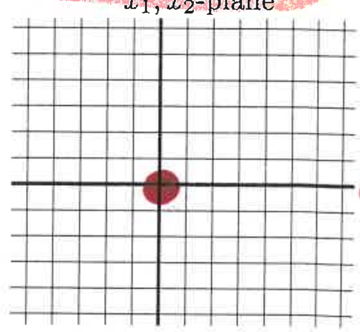
IVP: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ implies $c_1 = c_2 = 0$.

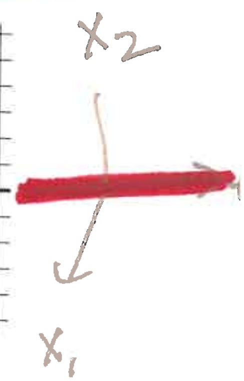
Thus IVP soln: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Hence $x_1 = 0$ and $x_2 = 0$

https://www.geogebra.org/3d (t, 0, 0)

x_1, x_2-plane





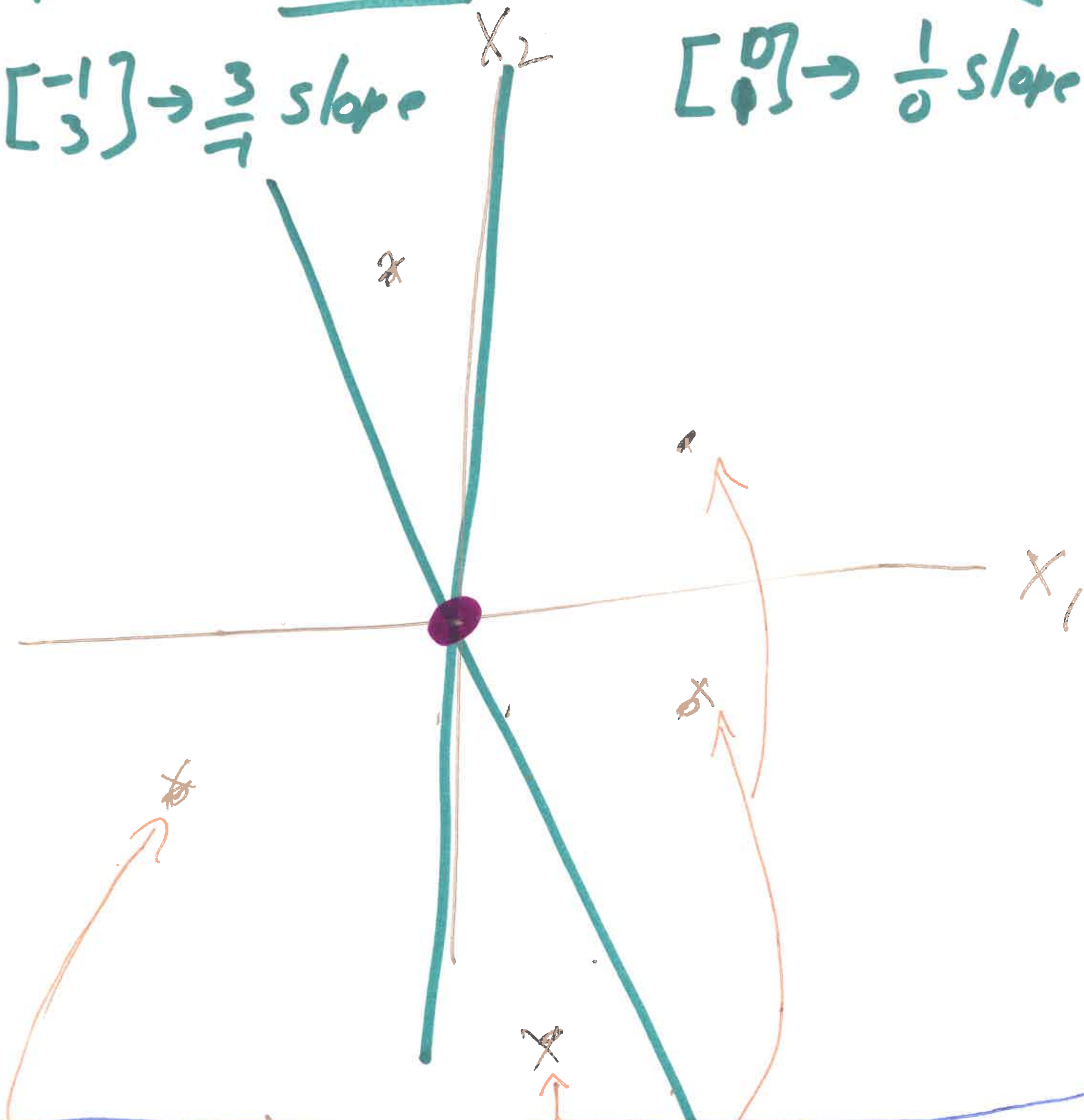
$x_1(t) = 0$, $x_2(t) = 0$ } constant soln

||

equilibrium soln

$\vec{x}' = A\vec{x} \Rightarrow \vec{x}(t) = 0$ is a soln $\Rightarrow \vec{x}(t)$ is an equil soln

Two <u>real</u> e. values



What if $c_1 \neq 0$ and $c_2 \neq 0$

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}\mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{-2t} or e^{5t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$.

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$?

Thus for large **positive** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ________________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ________________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{-2t} or e^{5t}?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$?

Thus for large **negative** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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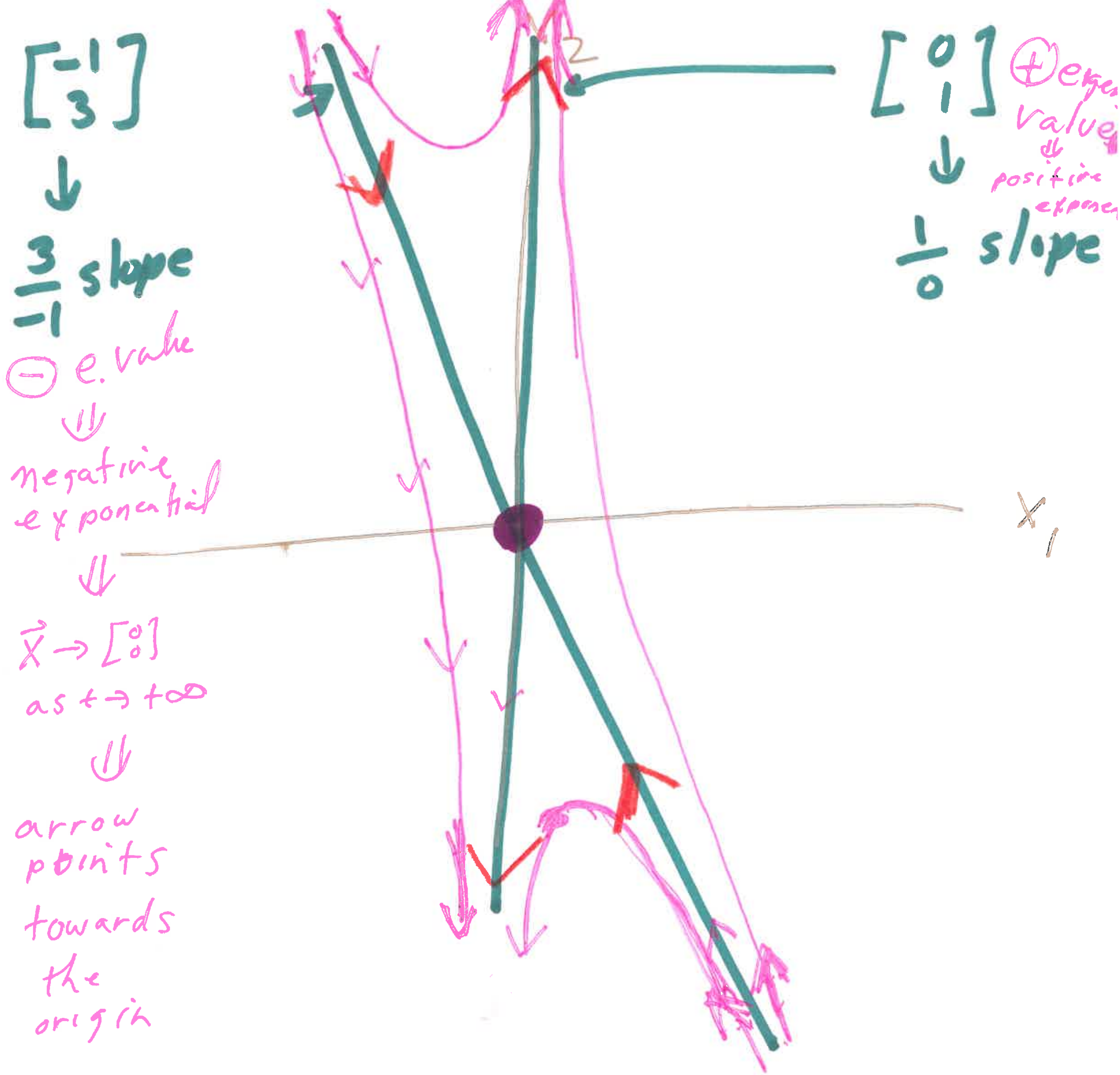
One + and one − e. value



Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ______. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ______. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t which is larger: e^{2t} or e^{5t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is
$\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$ or $c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t}$

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{2t} or e^{5t}?

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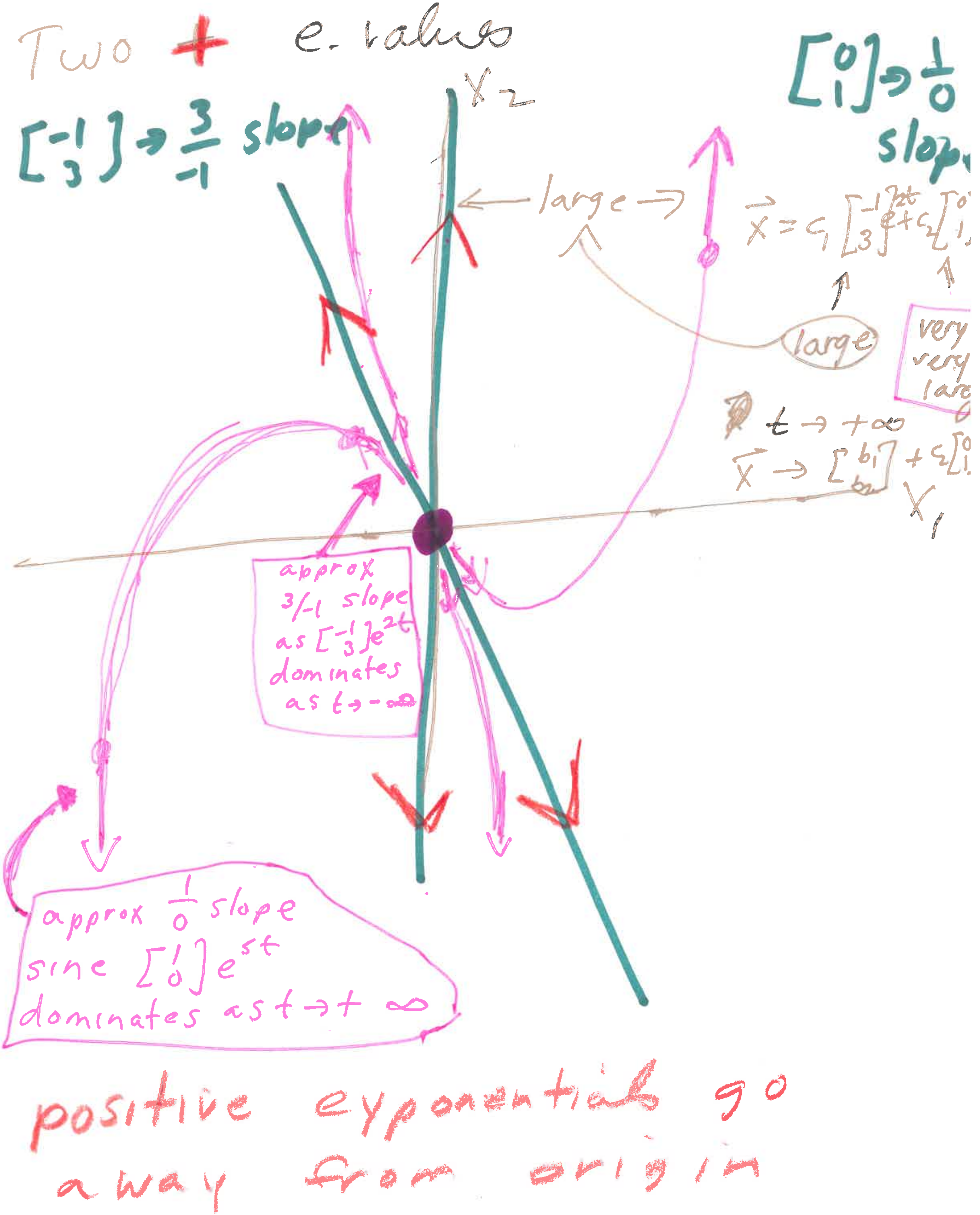

Two + e. values
$[{-1 \atop 3}] \to \frac{3}{-1}$ slope
$[{0 \atop 1}] \to \frac{1}{0}$ slope
X_2
large
$\vec{x} = c_1 [{-1 \atop 3}] e^{2t} + c_2 [{0 \atop 1}]$
large
very very large
$t \to +\infty$
$\vec{x} \to [{b_1 \atop b_2}] + c_2 [{0 \atop 1}]$
X_1
approx 3/-1 slope as $[{-1 \atop 3}] e^{2t}$ dominates as $t \to -\infty$
approx $\frac{1}{0}$ slope sine $[{1 \atop 0}] e^{5t}$ dominates as $t \to +\infty$
positive eyponentials go away from origin

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

For large **positive** values of t which is larger: e^{-5t} or e^{-2t}?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t}$ or $c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$?

Thus for large **positive** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
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* approaches the line $y = mx$ with slope $m =$ __________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ __________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1t}$ and $||c_2\mathbf{w}||e^{r_2t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: e^{-5t} or e^{-2t}?

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Thus for large **negative** values of t, such trajectories (where $c_1c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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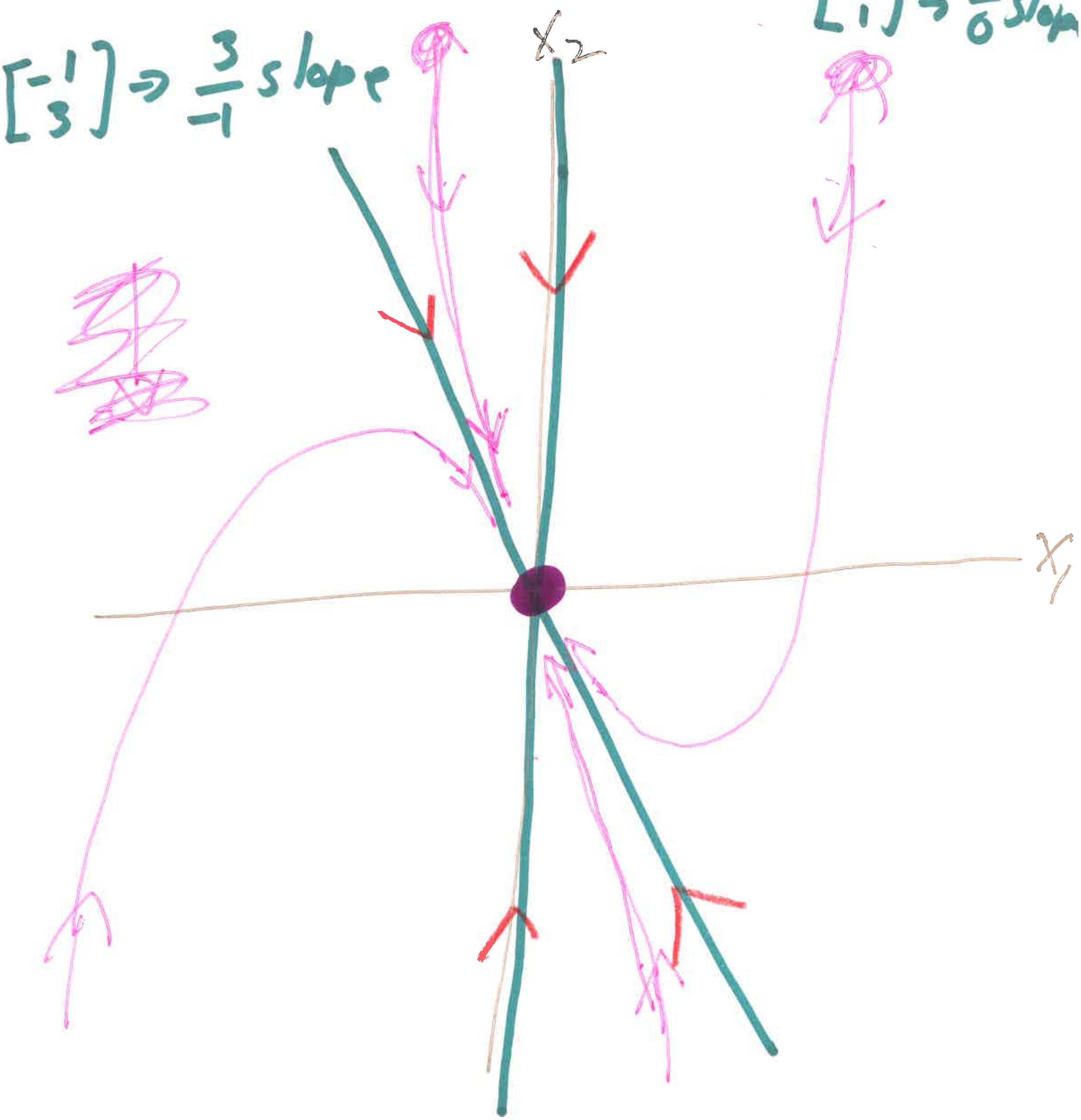
Two — e. values



negative exponentials go towards the origin

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1' = 2x_1$$

$$x_2' = 9x_1 + 5x_2$$

$$\frac{x_2'}{x_1'} = \frac{dx_2/\cancel{dt}}{dx_1/\cancel{dt}} = \boxed{\frac{dx_2}{dx_1} = \frac{9x_1+5x_2}{2x_1}}$$