3. Solve the initial value problem (where $t > 0$): $ty' + y = \sin t, \quad y(\frac{\pi}{2}) = 1.$

From $ty' + y = \sin t$, one can immediately go to the product rule step:

$(ty)' = sint$ Check product rule $(ty)' = ty' + y$.

If you didn't notice the product rule:

$1y' + \frac{1}{t}y = \frac{sint}{t}$

Integrating factor $u(t) = e^{\int \frac{dt}{t}} = e^{ln(t)} = t$

Multiply BOTH sides by integrating factor:

$ty' + y = \sin t$

$(ty)' = sint$

$\int (ty)'dt = \int sintdt$

$ty = -cos(t) + C.$

Plug in initial value, $y(\frac{\pi}{2}) = 1$:

$(\frac{\pi}{2})(1) = -cos(\frac{\pi}{2}) + C \Rightarrow (\frac{\pi}{2})(1) = 0 + C.$ Thus $C = \frac{\pi}{2}$

Hence $ty = -cos(t) + \frac{\pi}{2}$ and $y = -\frac{cos(t)}{t} + \frac{\pi}{2t}$

NOTE: normally I would take off for not simplifying a fraction within a fraction. I didn't this time, but will in the future.

Answer: $y = -\frac{cos(t)}{t} + \frac{\pi}{2t}$

4. Fill in the blank:

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a. A 50 gallon tank contains 3 grams of Kryptonite. 8 gallons of water containing 2 grams of Krypotonite is pumped into the tanke every minute, and 8 gallons of solution is drained from the tank every minute. Assuming a thorough mixing, setup the initial value problem that describes this process. **(Do not solve!)**

Diff. eqn: _____. Initial Value: _____.

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b. Calculate the Wronskian $W(t^{-\frac{1}{2}}, t^2) =$ _____.

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c. Give the **form** of the particular (nonhomogenous) solution with undetermined coefficients for

$$y'' + 5y' + 4y = 2\cos(t)$$

$Y(t) =$ _____ (Do not solve!)

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d. The general solution for $y'' - y = 0$ is _____

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e. The general solution for $y'' + 9y = 0$ is _____.