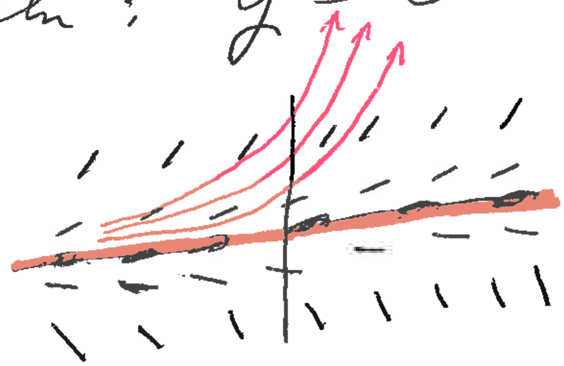
Homework 1b answers

1a) $y' = y$

Equilibrium soln: $y = 0$

Direction field:



Long term behaviour (depends only on y_0, not t_0)

If $y_0 > 0$, $y(t) \to +\infty$ as $t \to +\infty$

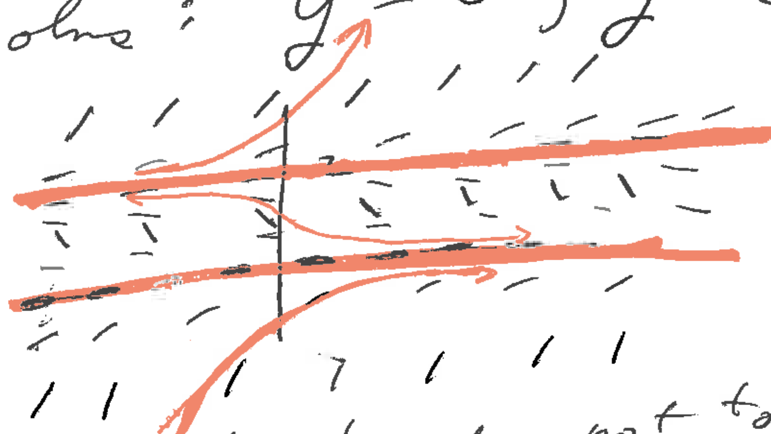
If $y_0 = 0$, $y(t) = 0 \to 0$ as $t \to +\infty$

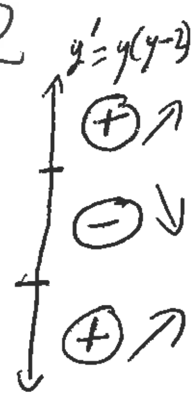
If $y_0 < 0$, $y(t) \to -\infty$ as $t \to +\infty$

1b) $y' = y(y-2)$

Equilibrium solns: $y = 0$, $y = 2$

Direction field:





Long term behaviour (depends only on y_0, not t_0)

If $y_0 > 2 \Rightarrow y(t) \to +\infty$ as $t \to +\infty$

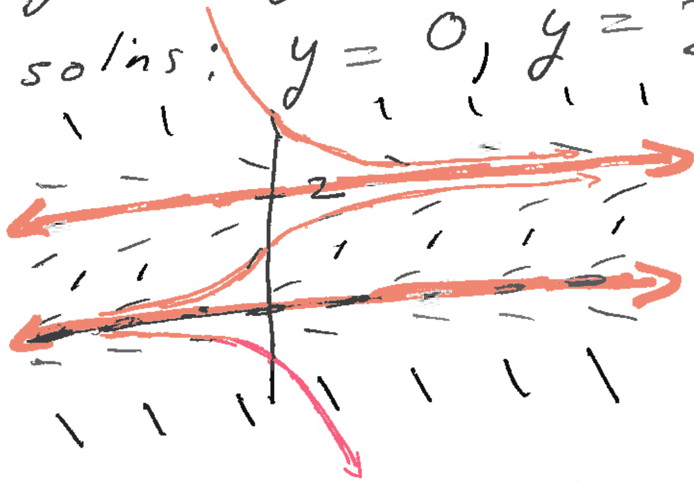
If $y_0 = 2 \Rightarrow y(t) = 2 \to 2$ as $t \to +\infty$

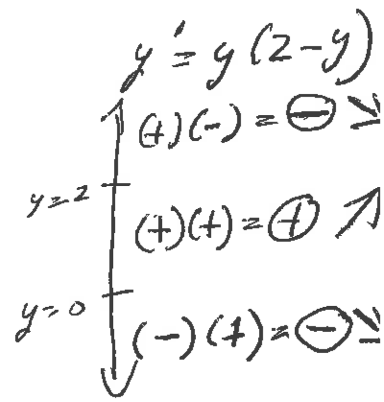
If $y_0 < 2 \Rightarrow y(t) \to 0$ as $t \to +\infty$

1c) $y' = y(2-y)$

Equilibrium solns: $y=0,\ y=2$

Direction field:





Long-term behaviour (depends only on y_0, not t_0)

If $y_0 > 0$, $y(t) \to 2$ as $t \to +\infty$

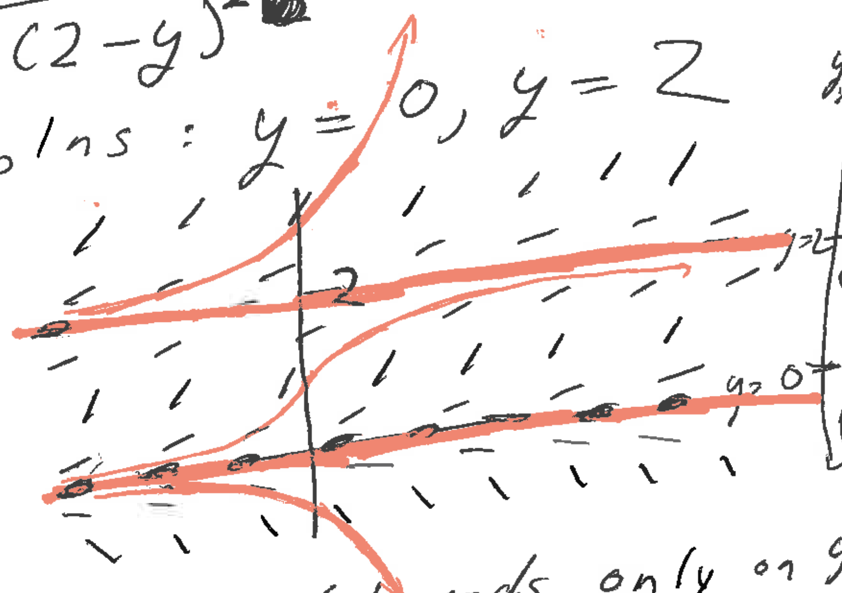
If $y_0 = 0$, $y(t) = 0 \to 0$ as $t \to +\infty$

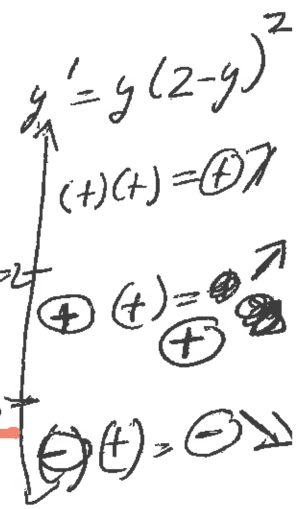
If $y_0 < 0$, $y(t) \to -\infty$ as $t \to +\infty$

1d) $y' = y(2-y)^2$

Equilibrium solns: $y=0,\ y=2$

Direction field:





Long-term behaviour (depends only on y_0, not t_0)

If $y_0 > 2$, $y(t) \to +\infty$ as $t \to +\infty$

If $0 < y_0 \leq 2$, $y(t) \to 2$ as $t \to +\infty$

If $y_0 = 0$, $y(t) = 0 \to 0$ as $t \to +\infty$

If $y_0 < 0$, $y(t) \to -\infty$

Note for problem 1, the slope only depended on y: $y' = f(y)$
Thus the long term behaviour of IVP with initial value (t_0, y_0) only depends on y_0 and not t_0

#2) $y' = t$

Direction field: