

$$1 \ y' + 3t^2 y = t^2, \quad \underline{y(0) = 2}$$

method 1: 2, 1 integrator factor

$$u(t) = e^{\int 3t^2 dt} = e^{t^3}$$

$$e^{t^3} y' + \underbrace{3t^2 e^{t^3}} y = t^2 e^{t^3}$$

pmv  $\int (e^{t^3} \cdot y)' dt = \int \frac{1}{3} (3t^2 e^{t^3}) dt$

Let  $u = t^3 \quad du = 3t^2 dt$

$$e^{t^3} \cdot y = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C$$

$$e^{-t^3} (e^{t^3} \cdot y) = \left( \frac{1}{3} e^{t^3} + C \right) e^{-t^3}$$

$$y = \frac{1}{3} + C e^{-t^3}$$

method 2:2.2 separation of variables  
1.2

$$y' + 3t^2 y = t^2 - 3t^2 y$$

$$\frac{dy}{dt} = t^2 - 3t^2 y$$

$$dt \frac{dy}{dt} = t^2 \left( \frac{1-3y}{1-3y} \right) dt$$

$$\int \frac{dy}{1-3y} = \int t^2 dt$$

↓ or use u-sub

$$\left[ -\frac{1}{3} \ln |1-3y| = \frac{1}{3} t^3 + C \right] \cdot (-3)$$

solve for y

$$e^{\ln |1-3y|} = e^{-t^3 + C}$$

$$|1-3y| = e^{-t^3} e^C$$

$$1-3y = Ce^{-t^3} - 1$$

$$\frac{-3y}{-3} = \frac{Ce^{-t^3}}{-3} - \frac{1}{-3}$$

$$y = Ce^{-t^3} + \frac{1}{3} \quad \leftarrow \text{general soln}$$

IVP soln:

$$y(0) = 2 : 2 = Ce^0 + \frac{1}{3}$$

$$\frac{5}{3} = \frac{6-1}{3} = 2 - \frac{1}{3} = C$$

$$y = \frac{5}{3} e^{-t^3} + \frac{1}{3} \quad \leftarrow \text{IVP soln}$$

$$\left[ y = \frac{2}{3} e + \sqrt{3} \right]^c \text{ s dm}$$