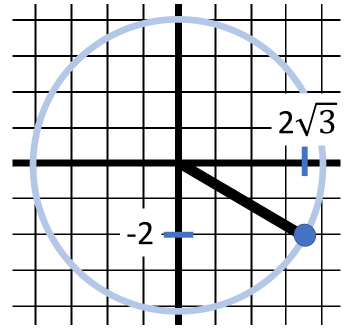
Quiz 3 Section 93 Oct 18, 2019

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$



$$u(t) = 2\sqrt{3}cos(2t) - 2sin(2t) = \underline{\hspace{6cm}}$$

[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = \underline{\hspace{9cm}}$$

where $u_1(t) =$ ______________________________ and $u_2(t) =$ ______________________________

[4] 3.) A spring mass system has a spring constant of 3N/m. A mass of 4kg is attached to the spring. The system is driven by an external force of sin(3t) N. The spring is streched 5 m and then set in motion with a downward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

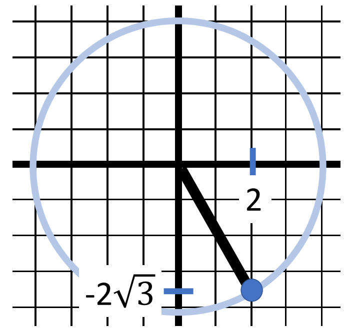
IVP: ____

[10] 4.) Solve $y'' - 4y = -12 + 8e^{2t}, \quad y(0) = 3,\ y'(0) = 6.$

Answer: ______________________________

Quiz 3 Section 93 Oct 18, 2019

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$



$$u(t) = 2cos(2t) - 2\sqrt{3}sin(2t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

where $u_1(t) =$ ____________________________ and $u_2(t) =$ ____________________________

[4] 3.) A 10 kg mass stretches a spring 5 m. The mass is acted on by an external force of 6cos(3t) N. The spring is compressed 4 m and then set in motion with an upward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: ____

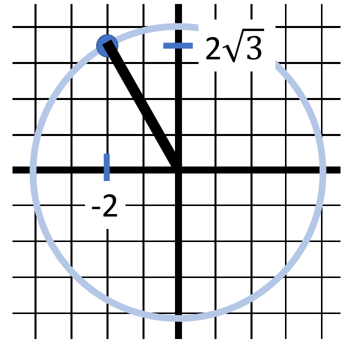
[10] 4.) Solve $y'' + 4y = 8 + sin(2t), \quad y(0) = 4,\ y'(0) = 1.$

Answer: ____

Quiz 3 Section 91 Oct 18, 2019

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$$u(t) = -2cos(2t) + 2\sqrt{3}sin(2t) = \underline{\hspace{6cm}}$$



[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = \underline{\hspace{9cm}}$$

where $u_1(t) =$ ____________________________ and $u_2(t) =$ ____________________________

[4] 3.) A spring is stretched 1m by a force of 2N. A mass of 9kg is attached to the spring and also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 1m/sec. The spring is set in motion from its equilibrium position with an upward velocity of 2m/s. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: ____

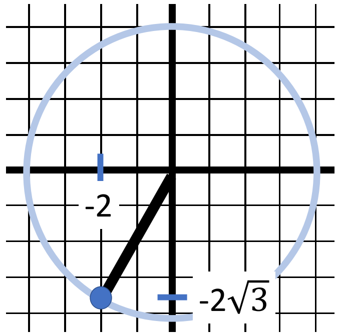
[10] 4.) Solve $y'' - y = t + 24e^{3t}, \quad y(0) = 5,\ y'(0) = 2.$

Answer: ____________________

Quiz 3 Section 91 Oct 18, 2019

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$$u(t) = -2cos(2t) - 2\sqrt{3}sin(2t) = \rule{5cm}{0.4pt}$$



[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = \rule{8cm}{0.4pt}$$

where $u_1(t) =$ ______________________________ and $u_2(t) =$ ______________________________

[4] 3.) A 20 kg mass stretches a spring 2 m. The mass is acted on by an external force of 9sin(5t) N and moves in a medium that imparts of viscous force of 8N when the speed of the mass is 4m/sec. The spring is compressed 4 m and then released. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: ____

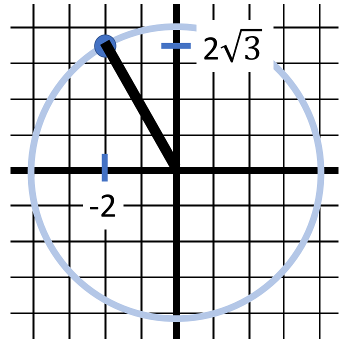
[10] 4.) Solve $y'' + 2y' + 5y = 50 + 34sin(2t), \quad y(0) = 4,\ y'(0) = 1.$

Answer: ____

Quiz 3 Section 91 Oct 18, 2019

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

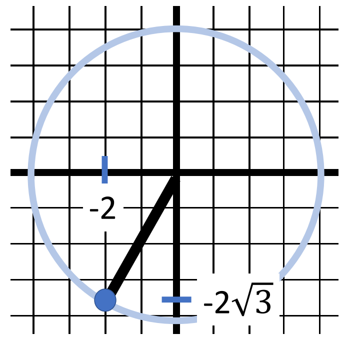
$$u(t) = -2cos(2t) + 2\sqrt{3}sin(2t) = \underline{\ 4cos(2t - \tfrac{2\pi}{3})\ }$$



$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

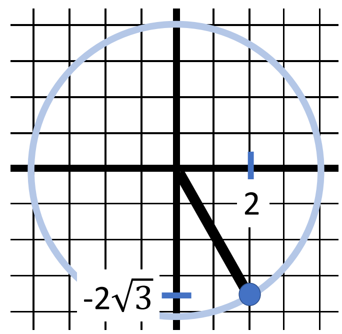
$$u(t) = -2cos(2t) - 2\sqrt{3}sin(2t) = \underline{\ 4cos(2t + \tfrac{2\pi}{3})\ }$$



$$-\frac{\pi}{2} - \frac{\pi}{6} = -\frac{4\pi}{6} = -\frac{2\pi}{3}$$

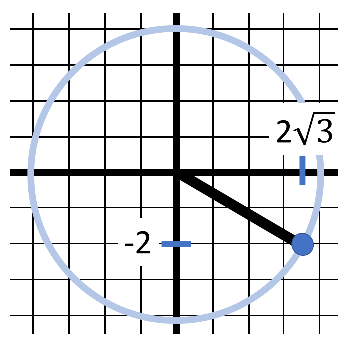
[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$$u(t) = 2cos(2t) - 2\sqrt{3}sin(2t) = \underline{\ 4cos(2t + \tfrac{\pi}{3})\ }$$



[3] 1.) Write the following in the form $u(t) = Rcos(bt - \delta)$

$$u(t) = 2\sqrt{3}cos(2t) - 2sin(2t) = \underline{\ 4cos(2t + \tfrac{\pi}{6})\ }$$



[3] 2.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are homogeneous solutions to $ay'' + by' + cy = g(t)$, then a non-homogeneous solution to this second order linear differential equation is

$$y(t) = u_1(t)\phi_1 + u_2(t)\phi_2$$

where $u_1(t) = \int \frac{\begin{vmatrix} 0 & \phi_2 \\ 1 & \phi_2' \end{vmatrix}}{\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix}} \frac{g(t)}{a} dt$ and $u_2(t) = \int \frac{\begin{vmatrix} \phi_1 & 0 \\ \phi_1' & 1 \end{vmatrix}}{\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix}} \frac{g(t)}{a} dt$

[4] 3.) A 10 kg mass stretches a spring 5 m. The mass is acted on by an external force of 6cos(3t) N. The spring is compressed 4 m and then set in motion with an upward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: $\underline{10u'' + 19.6u = 6cos(3t),\ u(0) = -4, u'(0) = -2}$

$mg = kL$, $k = \frac{10(9.8)}{5} = 2(9.8) = 19.6$

[4] 3.) A 20 kg mass stretches a spring 2 m. The mass is acted on by an external force of 9sin(5t) N and moves in a medium that imparts of viscous force of 8N when the speed of the mass is 4m/sec. The spring is compressed 4 m and then released. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: $\underline{20u'' + 2u' + 98u = 9sin(5t),\ u(0) = -4, u'(0) = 0}$

$mg = kL$, $k = \frac{20(9.8)}{2} = 98$

$8 = \gamma(4)$. Thus $\gamma = 2$

[4] 3.) A spring mass system has a spring constant of 3N/m. A mass of 4kg is attached to the spring. The system is driven by an external force of sin(3t) N. The spring is streched 5 m and then set in motion with a downward velocity of 2m/s. Assume that there is no damping. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: $\underline{4u'' + 3u = sin(3t),\ u(0) = 5, u'(0) = 2}$

[4] 3.) A spring is stretched 1m by a force of 2N. A mass of 9kg is attached to the spring and also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 1m/sec. The spring is set in motion from its equilibrium position with an upward velocity of 2m/s. State the initial value problem that describes the motion of this mass. **Do NOT solve**.

IVP: $\underline{9u'' + 3u' + 2u = 0,\ u(0) = 0, u'(0) = -2}$

$2 = 1k$. Thus $k = 2$

$3 = \gamma(1)$.

[10] 4.) Solve $y'' - 4y = -12 + 8e^{2t}, \quad y(0) = 3,\, y'(0) = 6.$

Step 1: Solve homogeneous $y'' - 4y = 0$

Guess $y = e^{rt}$: $\quad r^2 - 4 = 0$, Thus $r = \pm 2$

Homogeneous solution: $y = c_1 e^{2t} + c_2 e^{-2t}$

Step 2a: Find a non-homogeneous solution to $y'' - 4y = -12$

Guess $y = A$. Then $y' = 0$ and $y'' = 0$.

Plug in: $y'' - 4y = 12$: $\quad 0 - 4A = -12$.

Thus $A = 3$

Step 2b: Find a non-homogeneous solution to $y'' - 4y = 8e^{2t}$

Guess $y = Ate^{2t}$. Then $y' = Ae^{2t} + 2Ate^{2t}$ and $y'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t} = 4Ae^{2t} + 4Ate^{2t}$.

Plug in: $y'' - 4y = 8e^{2t}$: $\quad 4Ae^{2t} + 4Ate^{2t} - 4Ate^{2t} = 8e^{2t}$

$4Ae^{2t} = 8e^{2t}$. Thus $A = 2$.

General solution: $y = c_1 e^{2t} + c_2 e^{-2t} + 3 + 2te^{2t}$

$y' = 2c_1 e^{2t} - 2c_2 e^{-2t} + 2(e^{2t} + 2te^{2t})$

Initial values: $y(0) = 3,\, y'(0) = 6$

$y(0) = 3$: $\quad 3 = c_1 + c_2 + 3$. Thus $c_2 = -c_1$

$y'(0) = 6$: $\quad 6 = 2c_1 - 2c_2 + 2$. Thus $4 = 2c_1 + 2c_1$. Thus $4 = 4c_1$.

Hence $c_1 = 1$ and $c_2 = -c_1 = -1$

IVP solution: $y = e^{2t} - e^{-2t} + 3 + 2te^{2t}$

Answer: $\underline{y = e^{2t} - e^{-2t} + 3 + 2te^{2t}}$

[10] 4.) Solve $y'' - y = t + 24e^{3t}, \quad y(0) = 5,\ y'(0) = 2.$

Step 1: Solve homogeneous $y'' - y = 0$

Guess $y = e^{rt}$: $\quad r^2 - 1 = 0$, Thus $r = \pm 1$

Homogeneous solution: $y = c_1 e^t + c_2 e^{-t}$

Step 2a: Find a non-homogeneous solution to $y'' - y = t$

Guess $y = At + B$. Then $y' = A$ and $y'' = 0$.

Plug in: $-(At + B) = t$: $\quad -A = 1$ and $B = 0$.

So non-homogeneous solution to $y'' - y = t$ is $y = -t$

Sidenote: No y' term means we did not need constant term B, but including it is fine.

Step 2b: Find a non-homogeneous solution to $y'' - y = 24e^{3t}$

Guess $y = Ae^{3t}$. Then $y' = 3Ae^{3t}$ and $y'' = 9Ae^{3t}$.

Plug in: $y'' - y = 24e^{3t}$: $\quad 9Ae^{3t} - Ae^{3t} = 24e^{3t} \quad$ Thus$8Ae^{3t} = 24e^{3t}$ and $A = 3$.

General solution: $y = c_1 e^t + c_2 e^{-t} - t + 3e^{3t}$

$y' = c_1 e^t - c_2 e^{-t} - 1 + 9e^{3t})$

Initial values: $y(0) = 5,\ y'(0) = 2$

$y(0) = 5$: $\quad 5 = c_1 + c_2 - 0 + 3$. Thus $c_1 + c_2 = 2$

$y'(0) = 2$: $\quad 2 = c_1 - c_2 - 1 + 9$. Thus $c_1 - c_2 = -6$.

Thus $2c_1 = -4$ and thus $c_1 = -2$. Also $2c_2 = 8$ and $c_2 = 4$

IVP solution: $y = -2e^t + 4e^{-t} - t + 3e^{3t}$

Answer: $\underline{\ y = -2e^t + 4e^{-t} - t + 3e^{3t}\ }$

[10] 4.) Solve $y'' + 2y' + 5y = 50 + 34sin(2t), \quad y(0) = 4,\ y'(0) = 1.$

Step 1: Solve homogeneous DE: $y'' + 2y' + 5y = 0$

Guess $y = e^{rt}$: $\quad r^2 + 2r + 5 = 0$, Thus $r = \frac{-2\pm\sqrt{2^2-4(5)}}{2} = \frac{-2\pm\sqrt{-16}}{2} = \frac{-2\pm 4i}{2} = -1 \pm 2i$

Homogeneous solution: $y = c_1 e^{-t}cos(2t) + c_2 e^{-t}sin(2t)$

Step 2a: Find a non-homogeneous solution to $y'' + 2y' + 5y = 50$

Guess $y = A$. Then $y' = 0$ and $y'' = 0$.

Plug in: $y'' + 2y' + 5y = 50$: $\quad 0 + 5A = 50$. Thus $A = 10$

Step 2b: Find a non-homogeneous solution to $y'' + 2y + 5y = 34sin(2t)$

Guess $y = Acos(2t) + Bsin(2t)$.
Then $y' = -2Asin(2t) + 2Bcos(2t)$ and $y'' = -4Acos(2t) - 4Bsin(2t)$.

Plug in: $y'' + 2y + 5y = 34sin(2t)$:
$-4Acos(2t) - 4Bsin(2t) + 2(-2Asin(2t) + 2Bcos(2t)) + 5(Acos(2t) + Bsin(2t)) = sin(2t)$

$-4Acos(2t) + 4Bcos(2t) + 5Acos(2t) = 0$. Thus $-4A + 4B + 5A = 0$. Thus $A = -4B$

$-4Bsin(2t) - 4Asin(2t) + 5Bsin(2t)) = 34sin(2t)$.

Thus $-4B - 4A + 5B = 1$. Thus B - 4A = B - 4(-4B) = 17B = 34. Thus $B = 2$ and $A = -8$

General solution: $y = c_1 e^{-t}cos(2t) + c_2 e^{-t}sin(2t) + 10 - 8cos(2t) + 2sin(2t)$

$y' = c_1[-e^{-t}cos(2t) - 2e^{-t}sin(2t)] + c_2[-e^{-t}sin(2t) + 2e^{-t}cos(2t)] + 16sin(2t) + 4cos(2t)$

Initial values: $y(0) = 4$, $y'(0) = 1$.

$y(0) = 4$: $\quad 4 = c_1 + 10 - 8$. Thus $c_1 = 2$

$y'(0) = 1$: $\quad 1 = c_1[-1] + c_2[2] + 4$. Thus $1 = 2 + 2c_1$. Thus $c_2 =$ -1/2

IVP solution: $y = 2e^{-t}cos(2t) + 10 - 8cos(2t) + 2sin(2t)$ – 1/2 exp(-2t)sin(2t)

Answer: $\underline{y = 2e^{-t}cos(2t) + 10 - 8cos(2t) + 2sin(2t)}$– 1/2 exp(-2t)sin(2t)

[10] 4.) Solve $y'' + 4y = 8 + sin(2t), \quad y(0) = 4,\ y'(0) = 1.$

Step 1: Solve homogeneous $y'' + 4y = 0$

Guess $y = e^{rt}$: $\quad r^2 + 4 = 0$, Thus $r = \pm 2i$

Homogeneous solution: $y = c_1 cos(2t) + c_2 sin(2t)$

Step 2a: Find a non-homogeneous solution to $y'' + 4y = 8$

Guess $y = A$. Then $y' = 0$ and $y'' = 0$.

Plug in: $y'' + 4y = 8$: $\quad 0 + 4A = 8$. Thus $A = 2$

Step 2b: Find a non-homogeneous solution to $y'' + 4y = sin(2t)$

Guess $y = t(Acos(2t) + Bsin(2t))$.
Then $y' = t[-2Asin(2t) + 2Bcos(2t)] + Acos(2t) + Bsin(2t)$ and
$y'' = t[-4Acos(2t) - 4Bsin(2t)] + -2Asin(2t) + 2Bcos(2t) + -2Asin(2t) + 2Bcos(2t)$.

Thus $y'' = (4B - 4At)cos(2t) + (-4Bt - 4A)sin(2t)$

Plug in: $y'' + 4y = sin(2t)$:
$(4B - 4At)cos(2t) + (-4Bt - 4A)sin(2t) + 4[t(Acos(2t) + Bsin(2t))] = sin(2t)$

$(4B - 4At + 4tA)cos(2t) + (-4Bt - 4A + 4tB)sin(2t) = sin(2t)$

$(4B)cos(2t) + (-4A)sin(2t) = sin(2t)$

Thus $4B = 0$ and $B = 0$. Also $-4A = 1$ Thus $A = -\frac{1}{4}$

General solution: $y = c_1 cos(2t) + c_2 sin(2t) + 2 - \frac{1}{4} t cos(2t)$

$y' = -2c_1 sin(2t) + 2c_2 cos(2t) - \frac{1}{4} cos(2t) + \frac{1}{2} t sin(2t)$

Initial values: $y(0) = 4,\ y'(0) = 1$

$y(0) = 4$: $\quad 4 = c_1 + 2$. Thus $c_1 = 2$

$y'(0) = 1$: $\quad 1 = 2c_2 - \frac{1}{4}$. Thus $c_2 = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$.

Answer: $\underline{y = 2cos(2t) + \frac{5}{8} sin(2t) + 2 - \frac{1}{4} t cos(2t)}$