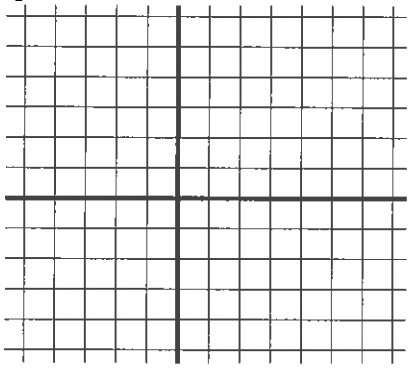
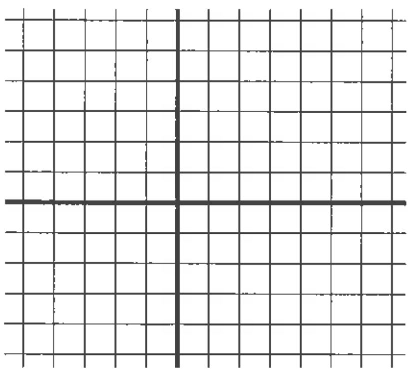
8.1 supplemental HW

1.) For each of the following differential equations (i) draw its direction field; (ii) sketch the solution of the direction field that passes through the point (-2, 1); (iii) state the general solution to the differential equation.

a.) $y' = 0$



b.) $y' = -1$



2.) Circle a solution to the differential equation whose direction field is given below:

FYI

A) ~~$y = t^2$~~

B) $y = \frac{1}{2}t + 1$

C) ~~$y = e^t$~~

D) $y = t + 1$

E) ~~$y = -2e^t$~~

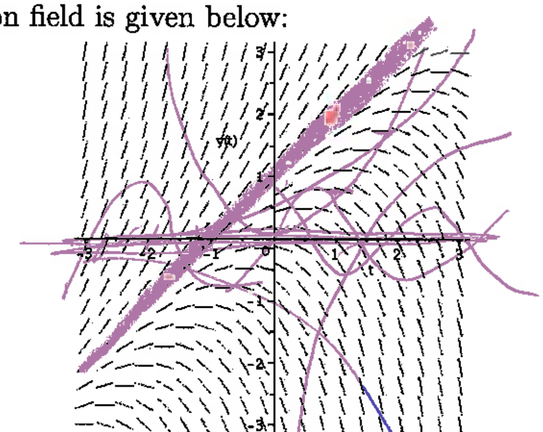
F) $y = 2t + 1$

G) ~~$y = ln(t)$~~

H) ~~$y = 0$~~

I) ~~$y = sin(t)$~~

J) ~~$y = cos(t)$~~



3.) Circle the differential equation whose direction field is given below:

A) ~~$y' = t^2$~~

B) ~~$y' = \frac{1}{2}t + 1$~~

C) ~~$y' = e^t$~~

D) ~~$y' = t + 1$~~

E) $y' = -2e^t$

F) $y' = y - t$

$y' = f(t, y)$

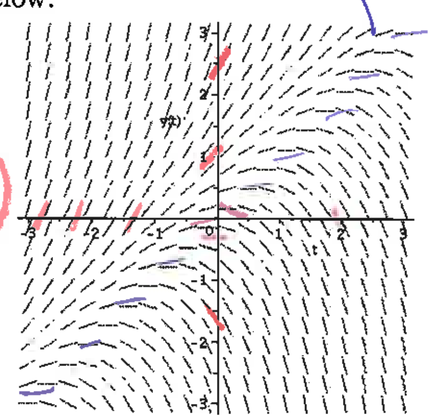
G) $y' = ln(t)$

H) ~~$y' = 0$~~

I) $y' = sin(t)$

J) $y' = cos(t)$

no equil



Quiz 1 Section 93 Sept 13, 2019

1.) Solve
$$t^2y' + 4ty = \frac{4}{t(t-1)(t+1)}$$

$t^2y' + 4ty = \frac{4}{t(t-1)(t+1)}$ Divide by t^2.

$y' + \frac{4}{t}y = \frac{4}{t^3(t-1)(t+1)}$ Note coefficient of y' is 1.

$y = e^{\int \frac{4}{t}dt} = e^{4ln|t|} = e^{ln(t^4)} = t^4$ Find integrating factor.

$t^4y' + 4t^3y = \frac{4t^4}{t^3(t-1)(t+1)}$ Multiply by integrating factor.

$(t^4y)' = \frac{4t}{(t-1)(t+1)}$ CHECK PRODUCT RULE!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

$\int (t^4y)'dt = \int \frac{4t}{(t-1)(t+1)}dt$ Integrate both sides.

We use partial fractions to integrate right-hand side to transform integral into the sum of two simpler integrals:

$t^4y = \int \frac{4t}{(t-1)(t+1)}dt = \int \frac{2}{t-1}dt + \int \frac{2}{t+1}dt = 2ln|t-1| + 2ln|t+1| + C$

Thus $y = 2t^{-4}ln|t-1| + 2t^{-4}ln|t+1| + Ct^{-4} = 2t^{-4}ln|(t-1)(t+1)| + Ct^{-4} = t^{-4}ln[(t^2-1)^2] + Ct^{-4}$

Partial fractions: $\frac{4t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$

Solve for A and B: $4t = A(t+1)+B(t-1) = At+A+Bt-B = (A+B)t+A-B.$

Note coefficient of t terms is 4. Thus $4 = A + B$

Note constant term is 0. Thus $0 = A - B$

$4 = 2A$. Thus $A = 2$ and $B = 2$

Alternatively, can plug in $t = 1, -1$ to determine A and B.

Note for this problem, we could instead use integration by substitution. Let $u = t^2 - 1$, then $du = 2tdt$

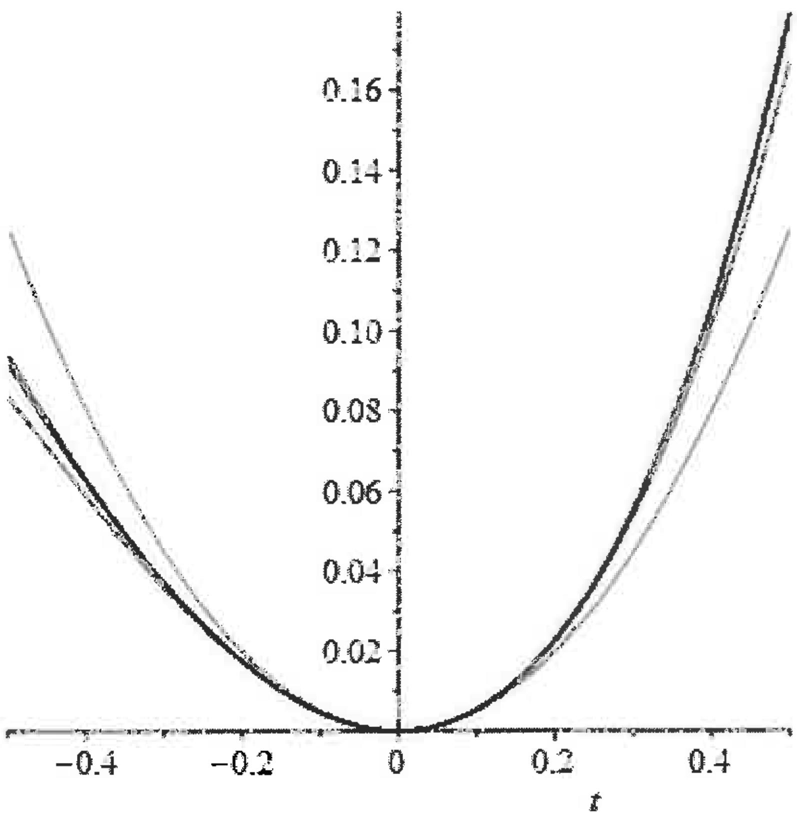
$\int \frac{4t}{(t-1)(t+1)}dt = \int \frac{4t}{t^2-1}dt = \int \frac{2du}{u} = 2ln|u| + C = 2ln|t^2-1| + C$

General Solution: $y = 2t^{-4}ln|t-1| + 2t^{-4}ln|t+1| + Ct^{-4}$ *or* $y = 2t^{-4}ln(t^2-1) + Ct^{-4}$ *or* [redacted]

2.8: Approximating soln to IVP using seq of fns.

$$\phi_0(t) = 0, \quad \phi_1(t) = \frac{t^2}{2}, \quad \phi_2(t) = \frac{t^2}{2} + \frac{t^3}{3},$$

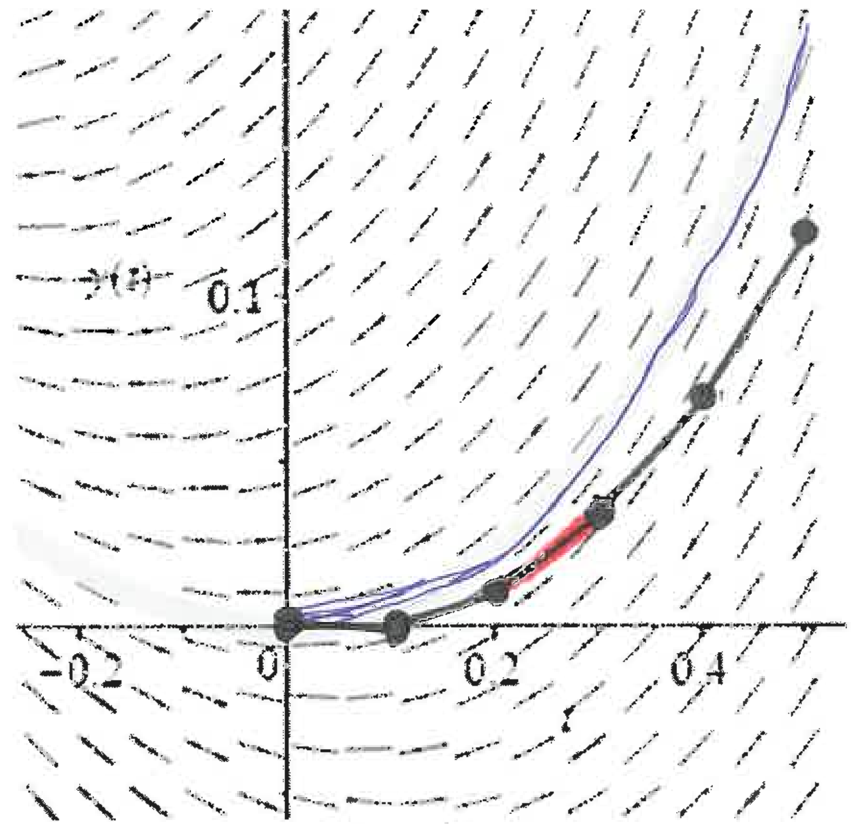
$$\phi_3(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6}, \quad \phi_4(t) = \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{6} + \frac{t^5}{15}$$



$y' = t + 2y \;,\; y(0) = 0$

2.7: Approximating soln to IVP using multiple tangent lines.

$$y(t) = \begin{cases} 0 & 0 \le t \le 0.1 \\ 0.1t - 0.01 & 0.1 \le t \le 0.2 \\ 0.22t - 0.034 & 0.2 \le t \le 0.3 \\ 0.364t - 0.0772 & 0.3 \le t \le 0.4 \\ 0.5328t - 0.14672 & 0.4 \le t \le 0.5 \end{cases}$$



$\Delta t = 0.1$

§3.1, 3, 4

Second order differential equation:

Linear equation with constant coefficients:
If the second order differential equation is

$$ay'' + by' + cy = 0,$$

then $y = e^{rt}$ is a solution

Need to have two independent solutions.

Solve the following IVPs:

1.) $y'' - 6y' + 9y = 0$ $\quad y(0) = 1,\ y'(0) = 2$

2.) $4y'' - y' + 2y = 0$ $\quad y(0) = 3,\ y'(0) = 4$

3.) $4y'' + 4y' + y = 0$ $\quad y(0) = 6,\ y'(0) = 7$

4.) $2y'' - 2y = 0$ $\quad y(0) = 5,\ y'(0) = 9$

$ay'' + by' + cy = 0$, $y = e^{rt}$, then
$ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$ implies $\boxed{ar^2 + br + c = 0}$,

Suppose $r = r_1, r_2$ are solutions to $ar^2 + br + c = 0$

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $r_1 \neq r_2$, then $b^2 - 4ac \neq 0$. Hence a general solution is $y = c_1e^{r_1t} + c_2e^{r_2t}$

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2 real soln

If $b^2 - 4ac > 0$, general solution is $y = c_1e^{r_1t} + c_2e^{r_2t}$.

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2 complex soln

If $b^2 - 4ac < 0$, change format to linear combination of real-valued functions instead of complex valued functions by using Euler's formula.

general solution is $y = c_1e^{dt}cos(nt) + c_2e^{dt}sin(nt)$
where $r = d \pm in$

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repeated

If $b^2 - 4ac = 0$, $r_1 = r_2$, so need 2nd (independent) solution: te^{r_1t}

Hence general solution is $y = c_1e^{r_1t} + c_2te^{r_1t}$.

Initial value problem: use $y(t_0) = y_0$, $y'(t_0) = y'_0$ to solve for c_1, c_2 to find unique solution.

§3.1, 3, 4

Second order differential equation:

Linear equation with constant coefficients:
If the second order differential equation is

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$ay'' + by' + cy = 0$, $y = e^{rt}$, then
$ar^2\cancel{e^{rt}} + br\cancel{e^{rt}} + c\cancel{e^{rt}} = 0$ implies $\boxed{ar^2 + br + c = 0}$,

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2 real soln's

If $b^2 - 4ac > 0$, general solution is $y = c_1e^{r_1t} + c_2e^{r_2t}$.

2 complex solns

If $b^2 - 4ac < 0$, change format to linear combination of real-valued functions instead of complex valued functions by using Euler's formula. simplified version

general solution is $y = c_1e^{dt}cos(nt) + c_2e^{dt}sin(nt)$ where $r = d \pm in$

If $b^2 - 4ac = 0$, $r_1 = r_2$, so need 2nd (independent) solution: te^{r_1t}

Hence general solution is $y = c_1e^{r_1t} + c_2te^{r_1t}$.

Initial value problem: use $y(t_0) = y_0$, $y'(t_0) = y'_0$ to solve for c_1, c_2 to find unique solution.

So why did we guess $y = e^{rt}$?

Goal: Solve linear homogeneous 2nd order DE with constant coefficients,

$$ay'' + by' + cy = 0 \text{ where } a, b, c \text{ are constants}$$

Standard mathematical technique: make up simpler problems and see if you can generalize to the problem of interest.

$y = e^{rt} \Rightarrow y' = re^{rt}$

Ex: linear homogeneous 1rst order DE: $y' + 2y = 0$

$r\cancel{e^{rt}} + 2\cancel{e^{rt}} = 0$

integrating factor $u(t) = e^{\int 2dt} = e^{2t}$

$r + 2 = 0$

$y'e^{2t} + 2e^{2t}y = 0$

$y = e^{2t}$ is a soln $\Leftarrow r = -2$

$(e^{2t}y)' = 0$. Thus $\int (e^{2t}y)'dt = \int 0dt$. Hence $e^{2t}y = C$

So $y = Ce^{-2t}$.

Thus exponential function could also be a solution to a linear homogeneous 2nd order DE

Guess $y = e^{rt} \Rightarrow y' = re^{rt} \Rightarrow y'' = r^2e^{rt}$

Ex: Simple linear homog 2nd order DE $y'' + 2y' = 0$.

$r^2\cancel{e^{rt}} + 2r\cancel{e^{rt}} = 0$

Let $v = y'$, then $v' = y''$

$r(r+2) = r^2 + 2r = 0$

$y'' + 2y' = 0$ implies $v' + 2v = 0$ implies $v = e^{2t}$.

$r = 0$

$y = e^{0t} = 1$

Thus $v = y' = \frac{dy}{dt} = Ce^{-2t}$. Hence $dy = Ce^{-2t}dt$ and

$$\boxed{y = c_1e^{-2t} + c_2e^{0t}}$$

$r = -2$

$y = e^{-2t}$