Quiz 4 SHOW ALL WORK
Nov 9, 2018

[15] 1.) Solve $ty' + 4y = t$

$1y' + \frac{4}{t}y = 1$

$u(t) = e^{\int \frac{4}{t}dt} = e^{4ln|t|} = e^{ln(|t|^4)} = t^4$.

Let $u(t) = t^4$

$t^4y' + 4t^3y = t^4$

$(t^4y)' = t^4$ **Check this step:** $(t^4y)' = t^4y' + 4t^3y$

$\int (t^4y)'dt = \int t^4dt$

$t^4y = \frac{t^5}{5} + C$

$y = \frac{t}{5} + Ct^{-4}$

Answer: $\underline{y = \frac{t}{5} + Ct^{-4}}$

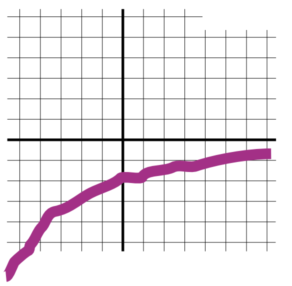
2.) Give that the solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} e^{-2t}$

Note c1 = 0 and c2 = 1 for this IVP.

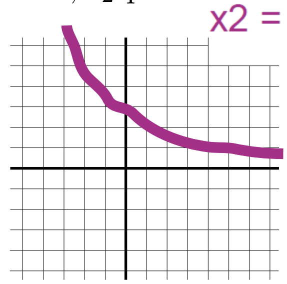
[7] 2a.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ in the

Thus $x1 = -2e^{-2t}$ and $x2 = 3e^{-2t}$

t, x_1-plane $x1 = -2e^{-2t}$



t, x_2-plane $x2 = 3e^{-2t}$

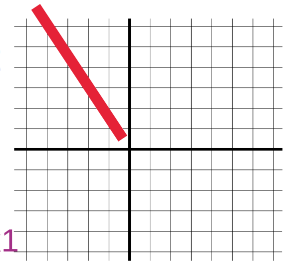


$x2 = 3e^{-2t}$

$x1 = -2e^{-2t}$

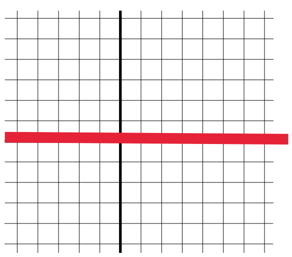
x2 = (3/-2)x1

x_1, x_2-plane

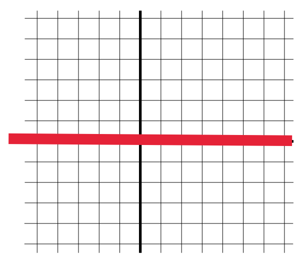


[3] 2b.) Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane

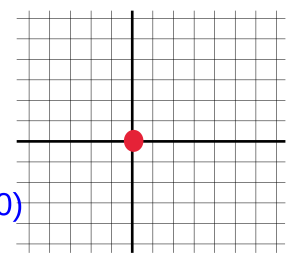


t, x_2-plane



Note c1 = 0 = c2 for this IVP. Thus x1 = 0 and x2 = 0 for all t. Thus we have the constant solution where (x1(t), x2(t)) = (0, 0) for all t

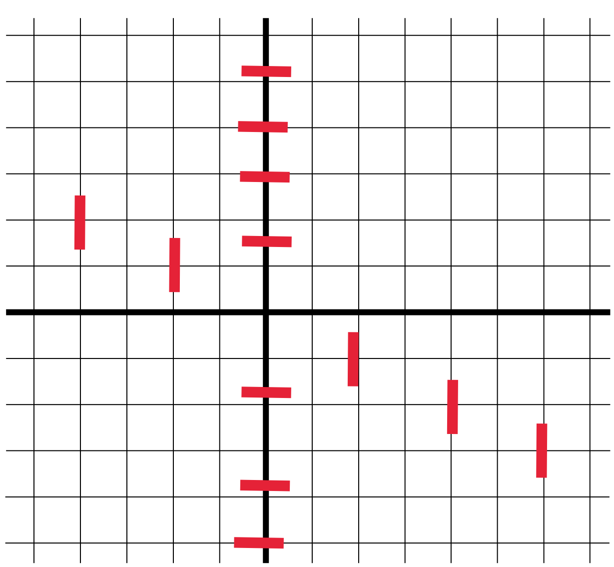
x_1, x_2-plane



[2] 2c.) The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

[3] 2d.) $\frac{dx_2}{dx_1} = \frac{3x1}{1x1 + 2x2}$

[2] 2e.) Plot several direction vectors where the slope is 0 and where slope is vertical.



[10] 2f.) Graph several trajectories.