

1.) The solution to $y'' + 16y = 36\cos(2t)$ is $y = c_1\cos(4t) + c_2\sin(4t) + 3\cos(2t)$

Use this fact to answer the following two questions.

[5] 1a.) Guess a possible non-homog soln for the following differential equation (do not solve): $y'' + 16y = 3\sin(4t) - e^{4t}$

Guess $y = t[A\sin(4t) + B\cos(4t)] + Ce^{4t}$

For explanations and examples,

see <http://homepage.divms.uiowa.edu/~idarcy/COURSES/100/3.5listSans.pdf>

and <http://homepage.divms.uiowa.edu/~idarcy/COURSES/100/3.5exA.pdf>

[3] 1b.) The general solution to $y'' + 16y = 36\cos(2t) + 32$ is

$$y = c_1\cos(4t) + c_2\sin(4t) + 3\cos(2t) + 2$$

See <http://homepage.divms.uiowa.edu/~idarcy/COURSES/100/FALL18/18.10.15.pdf>

2.) Circle T for true and F for false.

[2] 2a.) $L(f) = af'' + bf' + cf$ is a linear function on the space of all twice differentiable functions.

T

[2] 2b.) $L(f) = af'' + bf' + cf^2$ is a linear function on the space of all twice differentiable functions.

F

[2] 2c.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to $ay'' + by' + cy = 0$, $y = \psi_1(t)$ is a solution to $ay'' + by' + cy = g_1(t)$, and $y = \psi_2(t)$ is a solution to $ay'' + by' + cy = g_2(t)$, then the **general** solution to $ay'' + by' + cy = g_1(t) + g_2(t)$ is $y = c_1\phi_1(t) + c_2\phi_2(t) + \psi_1(t) + \psi_2(t)$.

F

[2] 2c.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to $ay'' + by' + cy = 0$, $y = \psi_1(t)$ is a solution to $ay'' + by' + cy = g_1(t)$, and $y = \psi_2(t)$ is a solution to $ay'' + by' + cy = g_2(t)$, then $y = c_1\phi_1(t) + c_2\phi_2(t) + \psi_1(t) + \psi_2(t)$ is also a solution to $ay'' + by' + cy = g_1(t) + g_2(t)$.

T

[2] 2c.) Suppose $y = \phi_1(t)$ and $y = \phi_2(t)$ are linearly **independent solutions** to $ay'' + by' + cy = 0$,

$y = \psi_1(t)$ is a solution to $ay'' + by' + cy = g_1(t)$, and

$y = \psi_2(t)$ is a solution to $ay'' + by' + cy = g_2(t)$, then the general solution to

$ay'' + by' + cy = g_1(t) + g_2(t)$ is $y = c_1\phi_1(t) + c_2\phi_2(t) + \psi_1(t) + \psi_2(t)$.

T

Note for 2c, I forgot to include linearly independent, so if lost 2 points because I graded your problem incorrectly, please let me know.

$$[2] \quad 2d.) \quad \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} = \sum_{j=0}^{\infty} (j+2)(j+1)a_{j+2}x^j = \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n$$

T

[2] 2e.) Suppose $f(x) = \sum a_n(x-3)^n$ has a radius of convergence $= r$ about the point $x_0 = 3$. Then we can define the domain of f to be $(3-r, 3+r)$.

T

[2] 2f.) Suppose $f(x) = \sum a_n(x+1)^n$ has a radius of convergence $= 4$ about the point $x_0 = -1$. Then we can define the domain of f to be $(-5, 3)$.

T