

[6] 1.) The Wronskian of  $e^{-t}\cos(t)$ ,  $e^{-t}\sin(t)$  is  $W(e^{-t}\cos(t), e^{-t}\sin(t)) =$  \_\_\_\_\_

[14] 2.) Solve  $t \frac{dy}{dt} + y = 8t^2$

Answer: \_\_\_\_\_

[14] 3.) Solve  $(3x^4 + 2y)dx + (2x + 4y^3)dy = 0$

Answer: \_\_\_\_\_

[18] 4.) Solve  $y''' - 4y' = 30e^{3t} + 8t$ ,  $y(0) = 0$ ,  $y'(0) = 6$ ,  $y''(0) = 16$ .

Answer: \_\_\_\_\_

[10] 5a.) Solve  $\mathbf{x}' = \begin{pmatrix} 4 & 5 \\ 4 & 3 \end{pmatrix} \mathbf{x}$

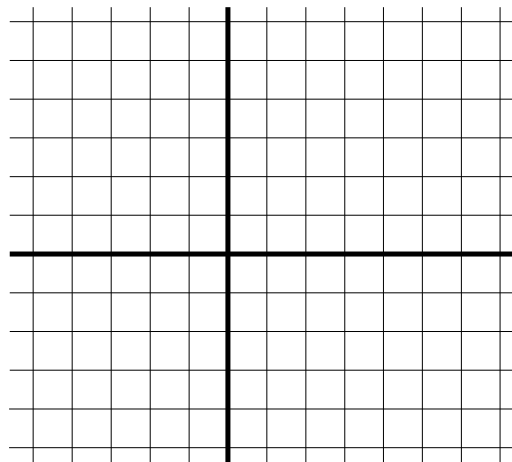
Answer: \_\_\_\_\_

[4] 5b.) Find all equilibrium solutions and determine whether the critical point is asymptotically stable, stable, or unstable. Also classify it as to type (nodal source, nodal sink, saddle point, spiral source, spiral sink, center).

Equilibrium solution: \_\_\_\_\_

Stability : \_\_\_\_\_ Type: \_\_\_\_\_

[4] 5c.) Plot a phase portrait for this system



[10] 6.) Given the recursive relation  $a_n = \frac{4a_{n-1}}{(n+1)n}$  where  $a_0 = 1$ ,

use induction to prove that  $a_n = \frac{4^n}{(n+1)!n!}$ .

7.) Given the differential equation:  $xy'' + 4y = 0$

[3] 7a.) Show that  $x = 0$  is a regular singular point of the above differential equation.

[4] 7b.) The indicial equation is \_\_\_\_\_

[1] 7c.) The roots of the indicial equation are \_\_\_\_\_

[5] 7d.) After plugging in the larger root, the recurrence relation is \_\_\_\_\_

[3] 7e.) For the larger root, if  $a_0 = 1$ , then  $a_1 =$  \_\_\_\_\_,  $a_2 =$  \_\_\_\_\_,  $a_3 =$  \_\_\_\_\_.

[4] 7f.) Use your answer to 7e to give an approximation of one solution to the above differential equation.

Answer for 7f: \_\_\_\_\_

