

Quiz 6 SHOW ALL WORK

Nov 30, 2018

[13] 1.) Find all equilibrium solutions and classify them (stable, asymptotically stable, semi-stable, unstable and if system of DEs, node, saddle, spiral, center). For the non-linear system of DEs, state all possibilities for type of equilibrium solution.

1a.) $y' = (y - 3)(y - 5)^8$ $y = 3$ is unstable, $y = 5$ is semi-stable.

1b.) $x' = y - 1, y' = (x - 3)y$

If $y - 1 = 0$, then $y = 1$

If $y = 1$, then $(x - 1)y = x - 3 = 0$. Thus $x = 3$.

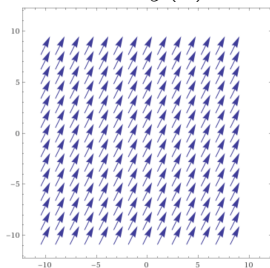
Jacobian matrix: $\begin{bmatrix} 0 & y \\ y & x - 3 \end{bmatrix}$

For $(x, y) = (1, 3)$, Jacobian matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Thus $(x(t), y(t)) = (1, 3)$ is a stable center or unstable spiral or asymptotically stable spiral.

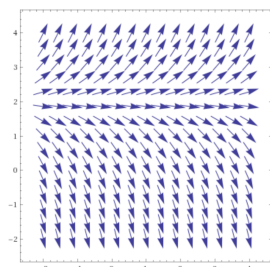
[7] 2.) The slope field for a first order differential equation is shown below. In addition to determining and classifying all equilibrium solutions (if any), also draw the trajectories satisfying the initial values $y(0) = 3$ and $y(1) = 0$.

2a.)



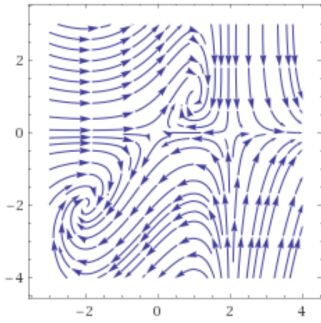
No equilibrium solution.

2b.)



Unstable equilibrium solution: $y = 2$.

[8] 3.) The stream plot in the x_1 - x_2 plane for a system of two first order differential equations is shown below. In addition to determining and classifying the 4 equilibrium solutions, also draw the trajectory satisfying the initial value $(x_1(0), x_2(0)) = (0, -2)$. Also describe the basins of attraction for each asymptotically stable equilibrium solutions.



$(x_1(t), x_2(t)) = (0, 0)$ is an unstable saddle.

$(x_1(t), x_2(t)) = (1, 2)$ is an asymptotically stable node.

basin of attraction: $x_1 > 0$.

$(x_1(t), x_2(t)) = (-1, 2)$ is an asymptotically stable node.

basin of attraction: $x_1 < 0$.

[5] 4.) Use Picard's iteration method to find a degree 3 polynomial approximation for the solution to the initial value problem, $y' = y + 6t^2$, $y(0) = 0$. Start with $\phi_0(t) = 0$.

$$\phi_1(t) = \int_0^t f(s, \phi_0(s)) ds = \int_0^t (\phi_0(s) + 6s^2) ds = \int_0^t (6s^2) ds = \int_0^t 2s^3 \Big|_0^2 = 2t^3$$

Answer: _____

[7] 5.) Using power series to find a degree 3 polynomial approximation for the general solution to $y' - y = 6x^2$ for x near 0

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} a_n n x^{n-1},$$

$$\sum_{n=1}^{\infty} a_n (n) x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 6x^2$$

$$\sum_{n=0}^{\infty} a_{n+1} (n+1) x^n - \sum_{n=0}^{\infty} a_n x^n = 6x^2$$

$$\sum_{n=0}^{\infty} [a_{n+1} (n+1) - a_n] x^n = 6x^2$$

$$a_{n+1} (n+1) - a_n = 0 \text{ for } n = 0, 1 \text{ and } n > 2$$

$$a_{n+1} = \frac{a_n}{n+1} \text{ for } n = 0, 1 \text{ and } n > 2$$

$$\text{For } n = 0: \quad a_1 = \frac{a_0}{1}$$

$$\text{For } n = 1: \quad a_2 = \frac{a_1}{2} = \frac{a_0}{2}$$

$$\text{For } n = 2: \quad a_3(3) - a_2]x^2 = 6x^2 \text{ implies } 3a_3 - a_2 = 6 \text{ and thus } a_3 = \frac{a_2+6}{3} = \frac{a_2}{3} + 2 = \frac{a_0}{6} + 2$$

Answer: _____