dy/dt derivative w.r.to t

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§2.1 Solve $t^3 y' + 3t^2 y = 4$ first order LINEAR ODE

NOTE PRODUCT RULE $\ddot{\cup}$ $\rightarrow$ $(t^3 \cdot y)' = 4$

$$\int (t^3 y)' \, dt = \int 4 \, dt$$

$$t^3 y = 4t + C$$

$$\Rightarrow \boxed{y = 4t^{-2} + Ct^{-3}}$$

§2.1 First order linear DE

$$y' + p(t) y = g(t)$$

Make product rule appear by using Integrating Factor:

$$u(t) = e^{\int p(t) \, dt}$$

Multiply both sides by $u(t) = e^{\int p(t)\,dt}$

$$1y' + p(t)y = g(t)$$

$$\left(e^{\int p(t)\,dt}\right) \cdot y' + \left(p(t)e^{\int p(t)\,dt}\right) \cdot y = g(t)e^{\int p(t)\,dt}$$

NOTE PRODUCT RULE

$$\left(e^{\int p(t)\,dt} \cdot y\right)' = g(t)e^{\int p(t)\,dt}$$

$$\int \left(e^{\int p(t)\,dt} \cdot y\right)' dt = \int g(t)e^{\int p(t)\,dt}\,dt$$

$$\left(e^{\int p(t)\,dt}\right)(y) = \int g(t)e^{\int p(t)\,dt}\,dt$$

$$y = e^{-\int p(t)\,dt} \cdot \int g(t)e^{\int p(t)\,dt}\,dt$$

Do not use formula

NOTE: BAD NOTATION only need one anti-derivative of $p(t)$

LINEAR, NOT SEPARABLE

Ex: $t^3y' + 3t^2y = 4$

If you don't notice it

$1y' + 3t^{-1}y = 4t^{-3}$

Let $u = e^{\int \frac{3}{t}dt} = e^{3\ln|t|+C}$

$= Ce^{\ln|t|^3} = C|t|^3$

Let $u(t) = t^3$ Choose any $u(t)$ that works

$1y' + 3t^{-1}y = 4t^{-3}$

Check this step

$t^3y' + 3t^2y = 4$

$\int (t^3y)'dt = \int 4\,dt$

$t^3y = 4t + C$

$y = 4t^{-2} + Ct^{-3}$

If you notice product rul

$\int (t^3y)'dt = \int 4\,dt$

$t^3y = 4t + C$

$y = 4t^{-2} + Ct^{-3}$

IVP Ex 2: $y' + 3t^2 y = t^2$, $y(0) = 2$

NOTE: both linear & separable thus can use either

2.1 integrating factor $e^{\int p(t)dt}$ to create product rule

or

2.2 separate variables

Method 2.1:

$$u(t) = e^{\int p(t)dt} = e^{\int 3t^2 dt} = e^{t^3}$$

$$1y' + 3t^2 y = t^2$$

$$e^{t^3} y' + 3t^2 e^{t^3} y = t^2 e^{t^3}$$

Check

$$\int (e^{t^3} y)'_{dt} = \int t^2 e^{t^3} dt$$

u-substitution Let $u = t^3$ $\frac{du}{3} = \frac{3t^2 dt}{3}$

$$e^{t^3} y = \int \frac{1}{3} e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{t^3} + C$$

Gen soln $y = \frac{1}{3} + Ce^{-t^3}$

IVP: $y(0) = 2$

$$2 = \frac{1}{3} + Ce^{0}$$

$$\frac{5}{3} = \frac{6}{3} - \frac{1}{3} = C$$

IVP soln: $y = \frac{1}{3} + \frac{5}{3}e^{-t^3}$

Method 2.2:

$$\frac{dy}{dt} + 3t^2 y = t^2$$

$$dy = (t^2 - 3t^2 y)\,dt$$

$$dy = (1-3y)t^2\,dt$$

$$\int \frac{dy}{1-3y} = \int t^2\,dt$$

or use u-sub

$$-\frac{1}{3}\ln|1-3y| = \frac{1}{3}t^3 + C$$

$$e^{\ln|1-3y|} = e^{-t^3+C}$$

$$1-3y = Ce^{-t^3}$$

$$y = Ce^{-t^3} + \frac{1}{3}$$

plug in initial value

Ex: $\frac{y'}{t} = \frac{-5y}{t} + 8t$

Need $1y' + p(t)y = g(t)$ format

$$y' = -5y + 8t^2$$

$$y' + 5y = 8t^2$$

$$u(t) = e^{\int 5\,dt} = e^{5t}$$

check! $\rightarrow$ $e^{5t}y' + 5e^{5t}y = 8t^2e^{5t}$

$$\int (e^{5t}y)'\,dt = \int 8t^2e^{5t}\,dt$$

$$e^{5t}y = \int 8t^2e^{5t}\,dt$$

$u = 8t^2 \qquad dv = e^{5t}$

$du = 16t \qquad v = e^{5t}/5$

$d^2u = 16 \qquad \int v = e^{5t}/25$

$$= \frac{8t^2e^{5t}}{5} - \left[\frac{16te^{5t}}{25} - \int \frac{16e^{5t}}{25}\,dt\right]$$

$$= \frac{8t^2e^{5t}}{5} - \frac{16te^{5t}}{25} + \frac{16e^{5t}}{125} + C$$

$$\boxed{y = \frac{8t^2}{5} - \frac{16t}{25} + \frac{16}{125} + Ce^{-5t}}$$