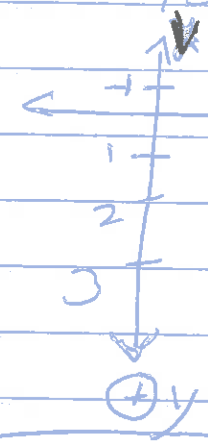
221
218
110

§ 1.1: Falling Ball

acts in direction opposite of motion } → air resistance = drag = γv ↑

acts downward } gravity mg ↓

Choose coordinate system:



we will choose positive direction pointing down

$F = ma = m\,dv/dt$

$F = mg - \gamma v$

$$m\frac{dv}{dt} = mg - \gamma v$$

positive direction pointing down

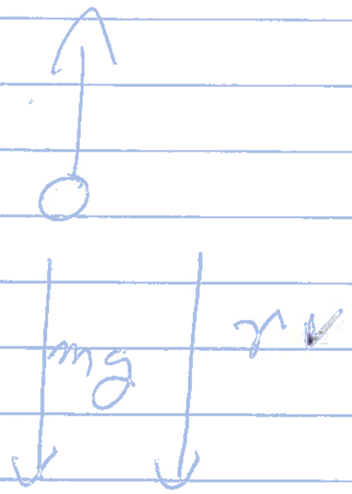
$\Rightarrow$ $+mg$ since gravity acts in positive direction

$\Rightarrow$ Falling ball $\Rightarrow$ ball travels in positive direction

$\Rightarrow v > 0 \Rightarrow$ air resistance acting upward $= -\gamma v$

Ball thrown up $\Rightarrow v < 0$

motion of ball is up = negative direction



$$m\frac{dv}{dt} = mg - \gamma v$$

} acts in positive direction

$v < 0 \Rightarrow \gamma v < 0$

$\Rightarrow -\gamma v > 0$

$$\Rightarrow \boxed{mv' = mg - \gamma v}$$

Suppose $m = 10$ kg, $\gamma = 2$ kg/sec

$$10v' = 10(9.8) - 2v$$

$$dv/dt = 9.8 - v/5$$

solve this diff eqn to determine how velocity changes over time

Alternatively using direction field

(2)

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

Direction field
= slope field
= graph of $\frac{dv}{dt}$ in (t, v) plane

t	v	dv/dt = slope (for this example = acceleration)
	60	$9.8 - \frac{60}{5} = 9.8 - 12$
	55	$9.8 - 55/5 = 9.8 - 11$
	49	0
	45	$9.8 - 45/5 = 9.8 - 9$
	40	$9.8 - 40/5 = 9.8 - 8$

for this example, $\frac{dv}{dt}$ does not depend on t

$dv/dt = 0 \Rightarrow$

$0 = 9.8 - \frac{v}{5}$

$\Rightarrow v = 5(9.8) = 49$

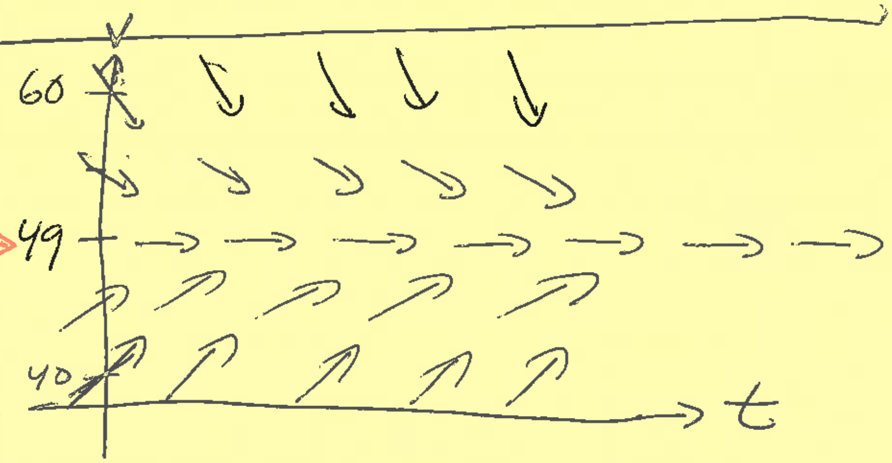
☆☆ Equilibrium sol'n = constant sol'n $\Leftrightarrow \frac{dv}{dt} = 0$

$v(t) = 49$
↖ constant

~~Large~~ $v > 49$
$\Rightarrow$ drag dominates
velocity ~~acceleration~~ decreases
object slows

constant velocity → 49

~~smaller~~ $v < 49$
gravity dominates
velocity ~~acceleration~~ increases
object increases in speed



☆☆ behaviour of v as $t \to \infty$

$v \to 49$ m/sec

§1.2 = 2.2 : Separation of Variables

Solve $\dfrac{dv}{dt} = 9.8 - \dfrac{v}{5}$

$\times\, 5\,dt$

$$5\,dv = (49 - v)\,dt$$

$$\frac{5\,dv}{49 - v} = dt$$

$$-5 \int \frac{1}{v - 49} = \int dt$$

$$-5 \ln|v - 49| = t + C$$

$$\ln|v - 49| = \frac{-t}{5} + C$$

Note C's are not the same constant

← we are sloppy and will let C swallow constants

$e \qquad e$

$$|v - 49| = e^{-t/5} e^{c} = Ce^{-t/5}$$

$$v - 49 = Ce^{-t/5}$$ ← C swallows $+/-$

$$\boxed{v = Ce^{-t/5} + 49}$$

(compare to handout figure)

Initial value Problem (IVP)

$$v' = 9.8 - v/5$$
$$v(t_0) = v_0$$

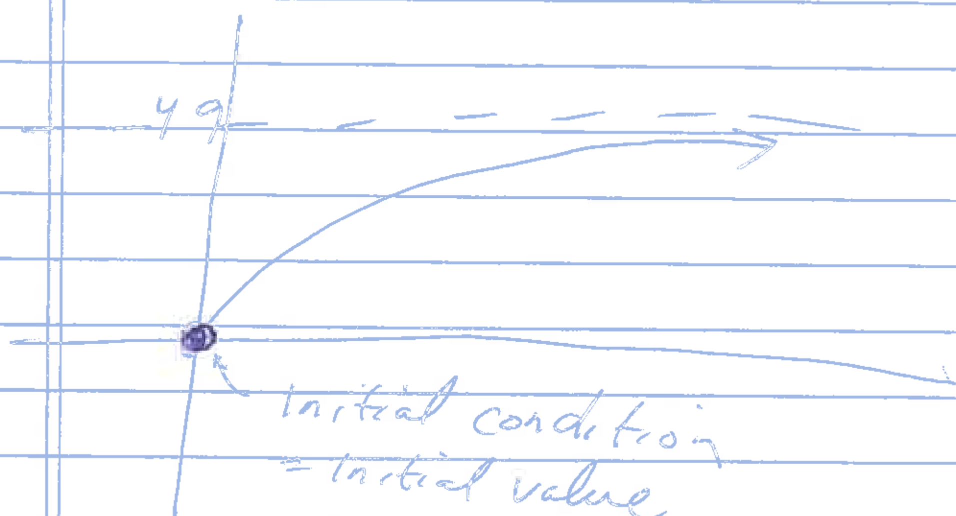
Ex: $v(0) = 0$

Use initial value to find C

$$v(t) = Ce^{-t/5} + 49$$

$$v(0) = 0 \Rightarrow 0 = Ce^{0} + 49$$
$$-49 = C$$

$$v(t) = -49e^{-t/5} + 49$$



compare to handout figure

at $t \to \infty$, $e^{-t/5} \to 0 \Rightarrow v \to 49$

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

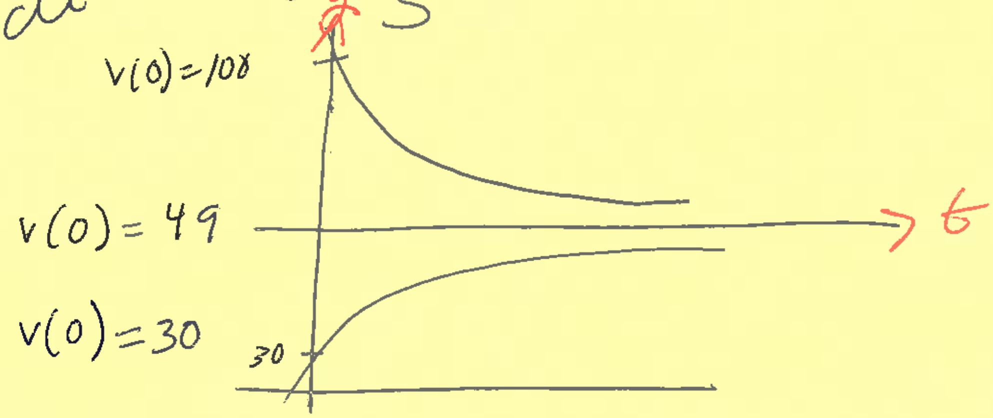
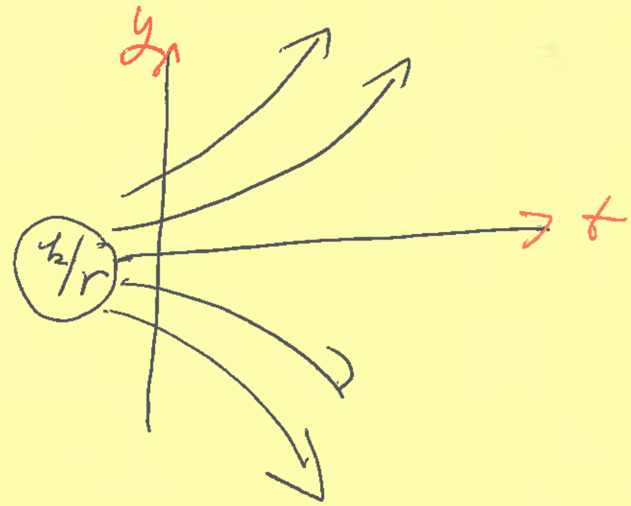


Fig 1.14

$$\frac{dp}{dt} = rp - h$$

$r > 0$



$0 = rp - h$

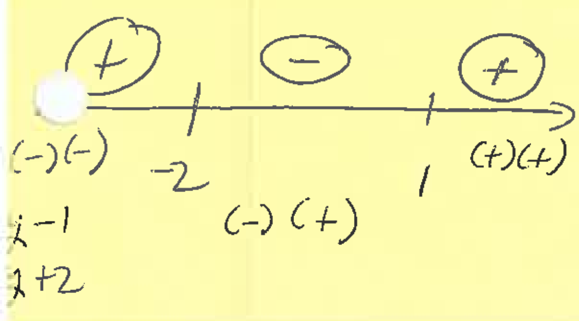
$p = \frac{h}{r}$ ← equilibrium soln

$$y' = (y-1)(y+2)$$

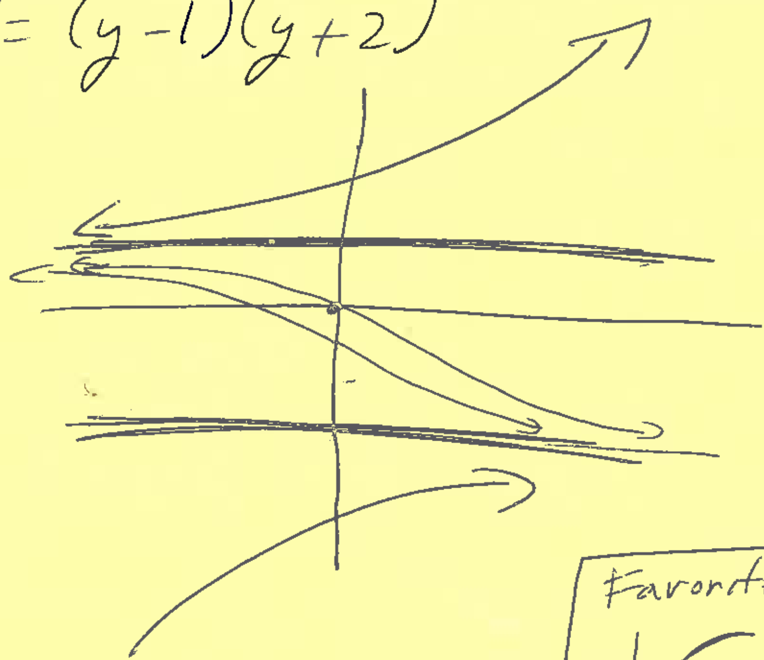
Equilibrium soln

$(y-1)(y+2) = 0$

$\Rightarrow y = 1, -2$



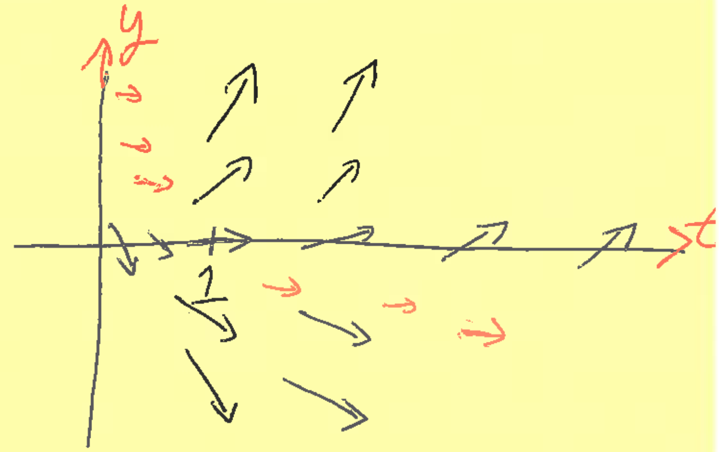
$y' = (y-1)(y+2)$



Favorite fn's

$y' = \ln t + y$

t	y	y'
1	y	y
t	0	ln t



No equilibrium sol'n

Pf: Suppose $y = c$ is a sol'n

$\Rightarrow y' = 0 \Rightarrow 0 = \ln t + y$

$\Rightarrow y = -\ln t$ not constant ✗

~~If $y = -\ln t \Rightarrow y' = $~~