Quiz 6 Form A, Dec 1, 2017

[10] 1a.) Find the general solution to the following differential equation system.

$$\mathbf{X}' = \begin{bmatrix} 1 & -1 \\ 6 & -4 \end{bmatrix} \mathbf{X}$$

$$\begin{bmatrix} 1-r & -1 \\ 6 & -4-r \end{bmatrix} \mathbf{X} = (1-r)(-4-r)+6 = r^2+3r-4+6 = r^2+3r+2 = (r+1)(r+2) = 0$$

$r = -1$: $\begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$r = -2$: $\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Check: $Av = rv$

$$\begin{bmatrix} 1 & -1 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer: $\mathbf{x} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-2t}$

[3] 1b.) Describe the behavior of the solution as $t \to \infty$: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

[7] 1c.) Sketch the phase portrait of this system (i.e. plot a few trajectories of the system).

See class notes (12/8)